MATH 6: EUCLIDEAN GEOMETRY 7
STRAIGHTEDGE AND COMPASS CONSTRUCTIONS

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Constructions with straightedge and compass

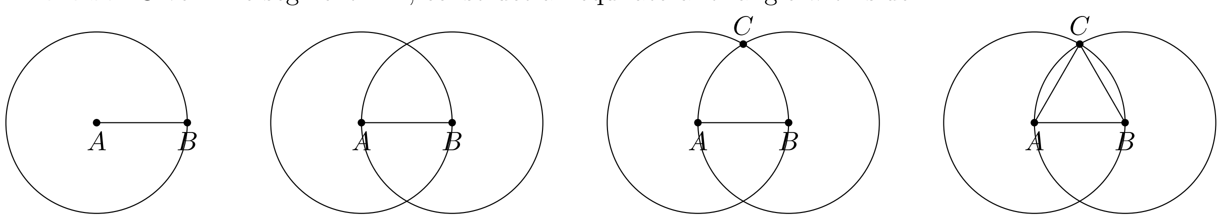
In geometry, there are two famous tools: a compass, which can draw circles, and a straightedge, which can draw lines. These tools have been used since ancient times to find ways to produce geometric figures without needing exact measurement tools. This process is called *straightedge-compass construction.* What's fascinating is that such simple tools can be used to construct all of the figures we have worked with so far, and plenty more. Let's see what we can do!

Here are the tools:

- Compass: given any point O and a line segment $\overline{AB}$, one can draw the circle centered at O with radius equal to the distance AB.
- Straightedge: given any two points A and B, one can draw the line $\overleftrightarrow{AB}$ or line segment $\overline{AB}$.
- Intersection points: Given any two figures, we can find and label their intersection points (this isn't exactly a tool, but still useful to state).

Whenever we construct something, though, we have to explain why it works. Here's an example:

Exercise: Given line segment $\overline{AB}$, construct an equilateral triangle with side $\overline{AB}$.



I did this by first drawing a circle centered at A with radius AB, using my compass. Then I made a circle centered at B with the same radius, using my compass. Then I labeled the point C, and used my straightedge to draw in line segments $\overline{CA}$ and $\overline{CB}$. Why does this construction work? Well, both the circles I drew have radius AB, and C is on both circles, so the distances CA and CB are both equal to the distance AB; therefore, this triangle is equilateral. Notice that, not only did I perform the steps of the construction, but I proved that my final result is indeed the one we're looking for.

Congruence tests for triangles

Recall the triangle congruence axioms. You will need to use them to explain why many of your constructions work.

Axiom 5 (ASA Rule)**.** *If* $\angle A = \angle A'$, $\angle B = \angle B'$ *and* $AB = A'B'$, *then* $\triangle ABC \cong \triangle A'B'C'$.

Axiom 6 (SSS Rule)**.** *If* $AB = A'B'$, $BC = B'C'$ *and* $AC = A'C'$ *then* $\triangle ABC \cong \triangle A'B'C'$.

Axiom 7 (SAS Rule)**.** *If* $AB = A'B'$, $AC = A'C'$ *and* $\angle A = \angle A'$, *then* $\triangle ABC \cong \triangle A'B'C'$.

Homework

1. Given two points A, B, construct the midpoint M of the segment AB.

2. (a) Given a line $\overleftrightarrow{AB}$ and a point P outside of $\overleftrightarrow{AB}$, construct a perpendicular to $\overleftrightarrow{AB}$ through P.
(b) Given a line $\overleftrightarrow{AB}$ and a point C on $\overleftrightarrow{AB}$, construct a perpendicular to $\overleftrightarrow{AB}$ through C.

3. Given an angle AOB, construct the angle bisector (i.e., a ray OM such that $\angle AOM \cong \angle BOM$)

4. Given line segment $\overline{AB}$, construct a square with side $\overline{AB}$.

5. Given three line segments with lengths a, b, c, construct a triangle with side lengths a, b, c. (The given line segments are disconnected, i.e. they don't already form a triangle.)

6. Given a circle, find its center. (You may label and use arbitrary points on the circle.)

7. Given a line $\overleftrightarrow{XY}$ and two points A and B that are not on $\overleftrightarrow{XY}$ but are on the same side of $\overleftrightarrow{XY}$, find a point C on $\overleftrightarrow{XY}$ such that $\angle XCA \cong \angle YCB$.

8. Given a circle and a point P outside this circle, construct the line through P which would be tangent to this circle (i.e., would touch it at exactly one point).

***9.** Given a line segment $\overline{AB}$, construct isosceles triangles AXB, AYB, and AZB all on the same side of $\overline{AB}$ such that $m\angle AXB = 90°$, $m\angle AYB = 60°$, $m\angle AZB = 45°$.

10. You have two fuses (specially treated cords, which burn slowly and reliably). Each of them would burn completely for one minute if lighted from one end. Using this, can you measure the time of 30 seconds? of 45 seconds? Note: some parts of the fuses may burn faster than others - so you can not just measure half of the fuse and say that it will burn for exactly 30 seconds.