GEOMETRY REVIEW PROBLEMS

1. Triangle Special Lines

Given any triangle $\triangle ABC$, there are three special types of lines that can be drawn inside the triangle; each type of line goes from one vertex to the opposite side, so we can for example define the three lines going from A to BC. Here they are: the median from A to BC is the line from A to the midpoint of BC; the altitude from A to BC is the line going through A that is perpendicular to BC; the angle bisector from A to BC is the line that goes through A such that the angle inside $\triangle ABC$ at vertex A is split in half by this line.

2. Homework

1. Let l be a line and P a point not on l. Let M be on l such that $PM \perp l$ and N be some point other than M on l. Prove that $PN > PM$ (you will need to use the theorem that bigger angles are opposite bigger sides in a triangle).

2. In triangle $\triangle ABC$ draw median AM. On the extension of AM, take point N such that $MN = AM$. Show that $AB = NC$ and $AC = BN$.

3. Suppose that in triangle $\triangle ABC$, the angle bisector from A to BC is the same line as the altitude from A to BC. Prove that $\triangle ABC$ is isosceles. Which side is the base?

4. Two circles with centers C and D intersect in A and B. Show that $AB \perp CD$.

5. Let $\angle AOB$ be a 45° angle such that $OA = OB$. Recall that there is a unique perpendicular line to OA that goes through A; let this line intersect $\overrightarrow{OB}$ at point X. Similarly, let the perpendicular to OB through B intersect $\overrightarrow{OA}$ at point Y.

(a) Draw a diagram of the above described scenario, and prove that X is farther from O than A is - that is, prove that $OX > OA$.

(b) Let AX and BY intersect at point C. Draw the line segment OC on your diagram, and then prove that $\angle COA \cong \angle COB$.

(c) Draw in line segment XY on your diagram and extend $\overrightarrow{OC}$ so that it intersects XY. Prove that $\overrightarrow{OC}$ intersects XY at the midpoint of XY. What will be the angle of intersection?

6. Let $\triangle ABC$ be a triangle and P any point on the line segment BC. Let E be on AB such that $PE \perp AB$, and M be on the extension of line $\overleftrightarrow{PE}$ such that $ME = PE$ (E will be outside $\triangle ABC$); similarly, let F be on AC such that $PF \perp AC$, and N be on the extension $\overleftrightarrow{PF}$ such that $NF = PF$. Prove that:

(a) $AN = AM = AP$

(b) $BM + CN = BC$

(c) $\angle BMA + \angle CNA = 180°$

7. Let $\triangle ABC$ be a triangle. Extend line BC to an infinite line, and take points M and N on $\overleftrightarrow{BC}$ so that the order is M, B, C, N and $MB = AB$, and $CN = AC$ (M, N will be outside $\triangle ABC$). If E and F are the middle points of AM and AN, and P is the intersection of EB and FC, prove that:

(a) $m(\angle BEM) = m(\angle BEA) = 90°$

(b) $PM = PA = PN$

(c) $\angle PMB = \angle PAB$, $\angle PNC = PAC$

***8.** Let $ABCD$ and $ABEF$ be parallelograms such that E, F are on the line $\overleftrightarrow{CD}$; let the diagonals $\overline{AC}$, $\overline{BD}$ intersect at M and $\overline{AE}$, $\overline{BF}$ intersect at N. Prove that $\overline{MN} \parallel \overline{AB}$.

***9.** Prove that a line and a circle cannot intersect at three different points.