MATH 6
MATH BATTLE!

1. You have nine cards, each of which are labeled with one of the numbers 1, 2, or 3 on the front and one of the numbers 1, 2, or 3 on the back; additionally, no two cards are the same.
You select four cards at random from these nine cards. If you are allowed to flip the cards, what is the probability that you can lay them on a table so that all three numbers 1, 2, 3 are showing?
*You can visualize the cards by drawing a 3x3 table, and then finding that the required condition is impossible if and only if the selected cards entirely miss one column and one row of the same index - there are three such selections of a row and column of the table and each leads to a single, distinct collection of four cards. Without a table, you can say that the required condition is impossible if and only if one of the numbers is entirely missing from the cards, and if this is the case then there are 2*2=4 possible cards you can make out of the two remaining numbers, which leaves you with only one choice of 4 cards. With either method, you get that the probability is 1-3/(9 choose 4) = 1-3/126 = 1-1/42 = 41/42*

2. Simplify $a \iff (b \iff (c \iff (a \iff (b \iff c))))$
There are a few ways to do this - a truth table would consist of 8 rows, which is not too bad, depending on your disposition, it just takes some care. Method 2 involves noticing that (a equivalent to x) simplifies to x if a is true and (not x) if a is false, so you can split the above statement into two cases and work with those; then you can do a similar argument for b, splitting into four cases in total. Method 3 involves proving that (x equivalent to (y equivalent to z)) is equivalent to (x xor y xor z), which allows you to simplify the problem greatly because xor is associative. Method 4 involves proving directly that iff is associative, though you have to be careful here, as (a iff b iff c) has a standard interpretation of (a iff b)and(a iff c) which is not what an associative triple-iff results to, but with rearrangement of parentheses you can write the original statement as (a iff a)iff(b iff b)iff(c iff c). In all methods, you will find that the statement is always true, so it simplifies to True.

3. (omitted for copyright)

4. (omitted for copyright)

5. The numbers 1 to 6 are written in a row. Can the signs + or - be put between them so that the sum will be 0?
No. The sum of these numbers is 15, an odd number (invariant). If we flip some of the plus signs to minuses in the sum, every flip subtracts 2 times a number from the sum, which is an even number. As you can only subtract even numbers in this way, you cannot make 0 from 15.

6. I have several pieces of candy that I am putting into bags. I take one piece and put it into a bag, and call this bag 1. Then I take a second piece and put it in a bag, but I also put inside this bag a copy of bag 1 - so this bag has two objects in it: one piece of candy, and one bag with one piece of candy in it; I call this bag 2. Then I make bag 3 by putting inside it one piece of candy, a copy of bag 1, and a copy of bag 2. I do this again for bag 4 - bag 4 has in it a piece of candy, a copy of bag 1, a copy of bag 2, and a copy of bag 3. If I were to unpack bag 4 to get all the pieces of candy out, how many bags would I pull out in total?
7. You can draw a diagram with circles representing bags and squares representing the candy pieces, this makes it easy to draw out on a piece of paper or on the board, and at that point you can directly construct bag 4 and count the circles. Alternatively, notice that the construction of bag n is equivalent to the following: take two copies of bag n-1, and pull all the bags out of one of them (i.e. remove the outer bag). Then put all of these into a bag to get bag n. From this you can see that the number of bags inside bag n is one more than twice the number of bags inside bag n-1. Start with 0, the next numbers you get are 1, 3, 7 (which are one less than the powers of 2).