Please provide sufficient details about how you solved the problem. More difficult problems are marked with a $*$. If unable to solve a problem, please present your thoughts and any partial solution.

1. Compute the following powers:

   (a) 5^4

   (b) 7^3

   (c) 4^{-2}

   (d) $(-2)^5$

2. Find each of the following products

   (a) $\left(3a^2\right) \times \left(5a^3\right)$

   (b) $\left(-5a\right) \times \left(-10a^2\right) \times \left(-2a^3\right)$

   (c) $\left(4a^2b\right) \times \left(3a^3b^3\right) \times \left(2ab^4\right) \times (-2)$

3. Simplify each of the following algebraic expressions, by opening parenthesis:

   (a) $7 \times (x + y)$

   (b) $x \times (3x - 5)$

   (c) $x \times (x + y)$

   (d) $7 \times (x + y + 2)$

4. Simplify the following algebraic expressions, by collecting **like terms**:

   (a) $3x + 5x^2 + 27x - 11x^2$

   (b) $21xy - 3x^2y + 17 - 11xy + 23 - x^2y$

   (c) $13xy - (5xy - 20)$

   (d) $(xy^2)^2 + (xy)^2 + 3(x^2y^4 + x^2y^2)$

5. Evaluate the following algebraic expressions for $x = 3$ and $y = 7$:

   (a) $2x + 3$

   (b) $x^2 + y^2$

   (c) $(x + y)^2$

   (d) $-x^2 - y^2 + 3$

   (e) $3x^3 y^2$

6. The volume of a rectangular cuboid is the product of its length, width and height. The length of a cuboid is 3 times its width, and its height is one half its length. If its width is w cm, find its volume.

7. (a) An ATM machine dispenses cash using \$20 and \$50 bills. What sum can one withdraw using this ATM?

   (b) An ATM machine in the imaginary country of Khiva dispenses cash using 15 tugrik and 35 tugrik (tugrik is the name of the local currency). What sums can one withdraw using this ATM?

8. * I climb half the steps in a staircase. Next, I climb one-third of the remaining steps. Then I climb one-eighth of the rest and stop to catch my breath. What is the smallest possible number of steps in the staircase?

9. *

   (a) Show that $(a + 1)(a - 1) = a^2 - 1$.

   (b) Without using a calculator, compute 1999×2001