SchoolNova, Math 5c
Homework 2
Factorials and Divisibility Tests
September 22, 2019

1. Use divisibility tests to determine if 12345 divisible by 3? by 5? by 9? by 11?

2. What is the remainder when $1 + 41 + 441 + 4441$ is divided by 4?

3. A package of plastic forks contains 16 forks. A package of plastic knives contains 12 knives. What is the smallest number of packages of each kind you have to buy to get exactly the same number of forks and knives?

4. *Twin primes* are primes of the form $(p, p+2)$ which differ by 2. Some examples are (a) $(5, 7)$ (b) $(11, 13)$. List 3 more pairs of twin primes.

5. Let us define primes of the form $(p, p+2, p+4)$. An example is $(3, 5, 7)$. Are there any other primes of this form? Explain.

6. Consider the product of all numbers from 1 to 25: $1 \times 2 \times \ldots \times 24 \times 25$. How many 3s are are there in the prime factorization of this number?

7.* Without multiplying all the terms, show that

   (a) $10! = 6! \, 7!$
   (b) $10! = 7! \, 5! \, 3!$
   (c) $16! = 14! \, 5! \, 2!$

8.* Can a number written with 10 0's, 10 1's and 10 2's be divisible by 3? Can it be a perfect square?

9.* Jane claims that if you take any two-digit number, write a zero after it, and then write the original number so that you get a five-digit number, then the result will always be a multiple of 7. For example, if your original number is 17, then the five-digit number is 17017. Is she right? Can you explain why?