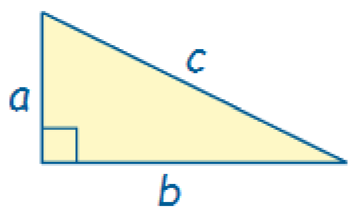
Math 5B: Classwork 24
Homework #24 is due April 19.

Pythagorean Theorem

Difference of squares formula: $(x-a)(x+a) = (x^2 - a^2)$
Square of the difference formula: $(a-b)(a-b) = (a-b)^2 = a^2 - 2ab + b^2$
Square of the sum formula: $(a+b)(a+b) = (a+b)^2 = a^2 + 2ab + b^2$

Area of the right triangle: $S_\Delta = \frac{1}{2} a \times b$

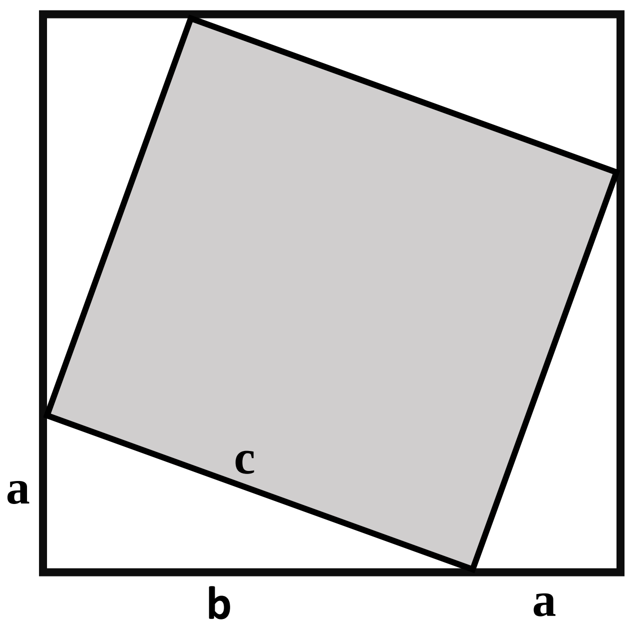


Theorem (Pythagorean theorem). In a right triangle with legs *a*, *b* and hypotenuse *c*, one has:

$$a^2 + b^2 = c^2$$

$$c = \sqrt{a^2 + b^2}$$

A proof of this theorem is illustrated below:



In this square, the ***total area*** is:

$$(a+b) \times (a+b) = a^2 + 2ab + b^2$$

Also, the area of each small triangle is $\frac{1}{2}ab$ and the area of the shaded area is c^2 such that the total area can also be written as:

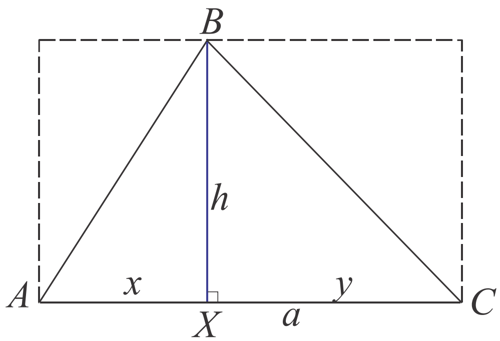
$$a^2 + 2ab + b^2 = 4 \times \frac{1}{2} ab + c^2$$
$$= 2ab + c^2$$
$$a^2 + b^2 = c^2$$

Homework

1. **Area of a triangle.**
   The area of a triangle is equal to half of the product of its height and the base, corresponding to this height.

   $S_\Delta = \frac{1}{2} h \times a$

a) Prove the above formula for the acute triangle:



Hint: find

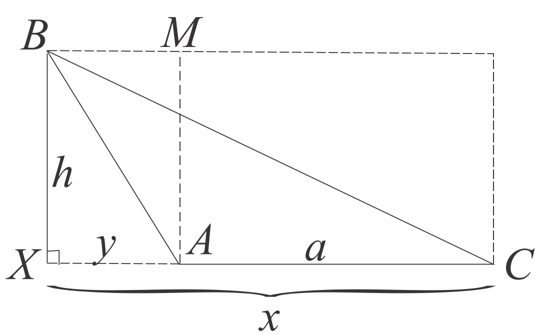
$$S_{\Delta ABX} = \dots., \quad S_{\Delta XBC} = \cdots,$$

Use

$$S_{\Delta ABC} = S_{\Delta ABX} + S_{\Delta XBC}$$

And apply distributive law.

b) Prove the above formula for the obtuse triangle:



Hint: find

$$S_{\Delta XBC} = \cdots, \qquad S_{\Delta XBA} = \cdots$$

Use :

$$S_{\Delta ABC} = S_{\Delta XBC} - S_{\Delta XBA} = \cdots$$

And apply distributive law.

2. **Constructions.**

Read "Introduction to Constructions" here -> https://www.mathopenref.com/constructions.html

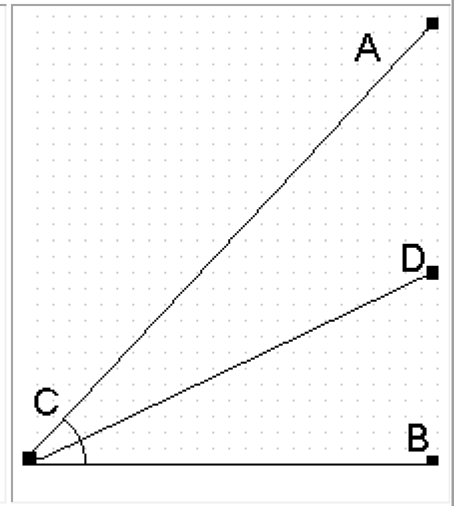
Given: a line l (el) and a point A that doesn't lie (belong) on (to) that line.

To do: Construct a line m (em) that goes through the point A and is perpendicular to the line l by following these directions -> https://www.mathopenref.com/printperpextpoint.html

Prove that line l is perpendicular to line m by using the properties for congruent triangles.

3. Construct an angle bisector

In the diagram to the right, the ray CD is the bisector of the angle ACB if and only if the angles ACD and BCD have equal measures.



Definition: The Bisector of an angle is a ray whose end point is the vertex of the angle and which splits the angle onto two equal angles.

Assignment: Construct an angle bisector for angle ACB (above) using directions below. Prove your construction is correct by using the properties for congruent triangles.

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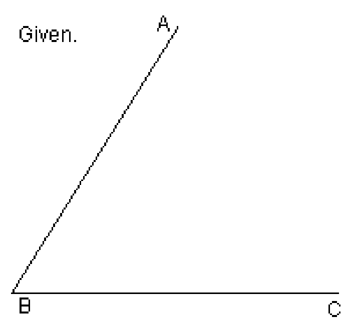
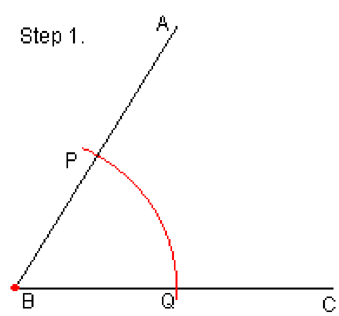
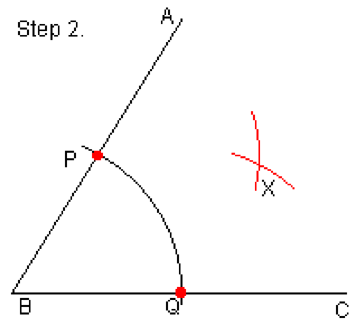
You are given an angle. Let's call it ABC.

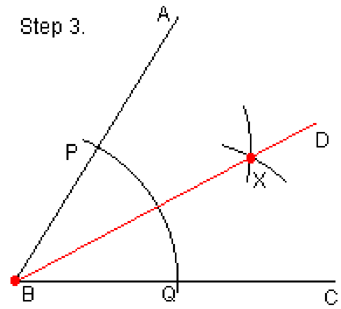


Step 1. Draw an arc that is centered at the vertex of the angle. This arc can have a radius of any length. However, the arc must intersect both sides of the angle. Let's call these intersection points **P** and **Q**. This provides a point on each line that is an equal distance from the vertex of the angle.



Step 2. Draw two more arcs. The first arc must be centered on one of the two points **P** or **Q**. It can have any length radius. The second arc must be centered on whichever point (P or Q) you did NOT choose for the first arc. The radius for the second arc MUST be the same as the first arc. Make sure you make the arcs long enough so that these two arcs intersect in at least one point. Let's call this intersection point **X**. Every intersection point between these arcs (there can be at most 2) will lie on the angle bisector.



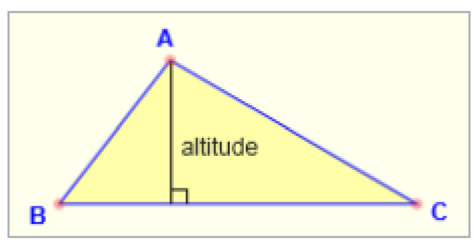


Step 3. Draw a line that contains both the vertex and **X**. Since the intersection points and the vertex all lie on the angle bisector, we know that the line which passes through these points **must** be the angle bisector.

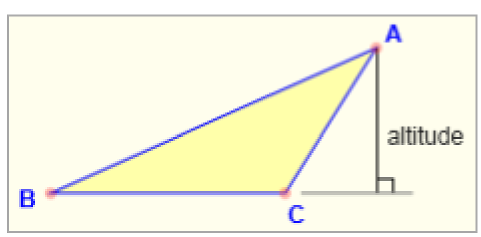
4. **Construct altitude of a triangle**

Using -> https://www.mathopenref.com/printaltitude.html construct an altitude of a triangle and prove it's perpendicular to the base. For proof rely on #2 from this homework.

Consider two cases:



a) When altitude is inside the triangle :



ANGLE C IS OBTUSE

b) When altitude is outside the triangle:

5. Construct a median (segment that divides the opposite side in half) of a triangle by following directions -> https://www.mathopenref.com/printmedian.html . Provide proof that the segment constructed by you is indeed the median of the given triangle. Prove that triangle CQJ is congruent to triangle CPJ. Which rule did you use?

   Hint:
   1. Name newly created (by intersecting circles drawn from Q and P) point at the top of the picture as C, and at the bottom as J.
   2. Proof that triangles CQJ and CPJ are congruent. Which rule did you use?
   3. Proof that triangles QJS and PJS are congruent. Which rule did you use?
   4. Then …