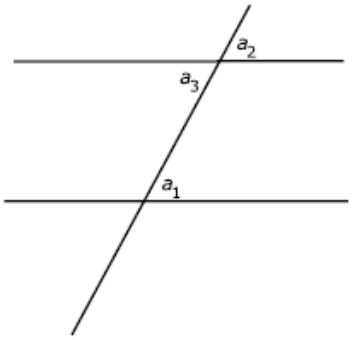
Math 5b: Classwork 23
Homework #23 is due on April 5th.

Parallel lines and alternate angles



If two parallel lines are intersected by a third line as shown in the figure to the right, then angles labeled by letters a_1 and a_3 (alternate interior angles) will be equal. Conversely, if these two angles are equal, then the lines must be parallel.

Congruence tests for triangles

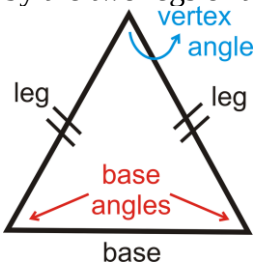
Rule 1 (Side-Side-Side). *If three sides of one triangle are equal to corresponding sides of another triangle, then the triangles are congruent.*
Rule 2 (*Angle-Side-Angle*). If two angles and a side between them of one triangle are the same as two angles and the side between them in another triangle, then the triangles are congruent.
Rule 3 (*Side-Angle-Side*). If two sides and an angle between them of one triangle are the same as two sides and an angle between them in another triangle, then the triangles are congruent.

Sum of angles of a polygon

Sum of angles of a triangle is 180^0. Sum of angles of an n-gon is $(n-2)\times 180^0$. For example, for a pentagon we get $3\times 180^0 = 540^0$.

Isosceles triangle

Definition: An isosceles triangle is a triangle that has at least two congruent sides.
The congruent sides of the isosceles triangle are called the legs. The other side is called the base and the angles between the base and the congruent sides are called base angles. The angle made by the two legs of the isosceles triangle is called the vertex angle.



Definition**: Median of a triangle** - A line segment from a vertex (corner point) to the midpoint of the opposite side.

Definition: **Angle bisector** - A line that splits an angle into two equal angles.

Definition: **Altitude of a triangle** - a segment that passes through a vertex of a triangle and meets the opposite side at the right angle.

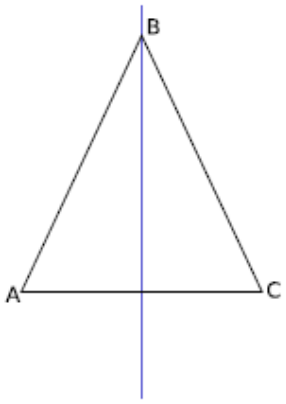
Let *ABC* be a triangle in which two sides are equal: AB = BC (such a triangle is called *isosceles*). Let M be the midpoint of the side AC, i.e. AM = M C.

Prove that: Angles ∠A and ∠C (base angles are equal).BM is the bisector

3) BM is the altitude

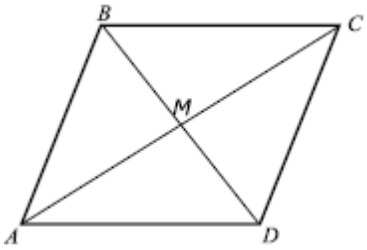
Some hints:

1. Triangles ΔABM and ΔCBM are congruent.
2. Angles ∠A and ∠C are equal.
3. ∠AMB = 90^0.



Parallelograms

Definition. A parallelogram is a quadrilateral in which opposite sides are parallel.



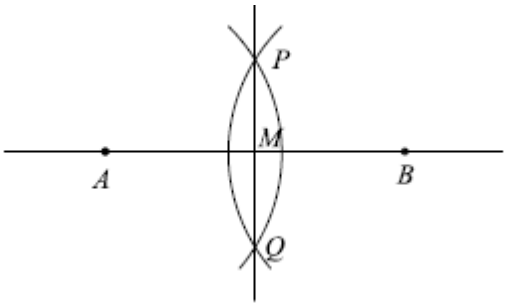
Properties:

1. In a parallelogram, opposite sides are equal. Conversely, if in a quadrilateral sides are equal, then it is a parallelogram.
2. in a parallelogram, diagonally opposite angles are equal ∠A = ∠C, ∠B = ∠D
3. The intersection point M of the two diagonals is the midpoint for each of them.

Homework

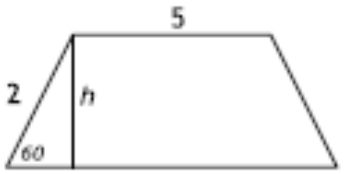
1. The following method explains how one can find the midpoint of a segment AB using a ruler and a compass:
    a. Choose radius r (it should be large enough) and draw circles of radius r with centers at A and B.
    b. Denote the intersection points of these circles by P and Q. Draw the line PQ.

c. Let M be the intersection point of lines PQ and AB. Then M is the midpoint of AB.



Draw this figure using the above method. Can you justify this method, i.e., prove that so constructed point will indeed be the midpoint of AB? You can use the defining property of a circle: for a circle of radius r, the distance from any point on this circle to the center is exactly r. [Hint: APBQ is a rhombus, what do you know about the diagonals?]

2. The figure to the right shows some of the angles and sides in a trapezoid. The height h of this trapezoid is equal to √3. Find all other angles, sides and area of the trapezoid. (Hint: you will need the Pythagorean theorem!)

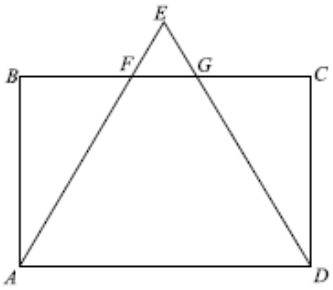


3. Solve the equations:

a) $|4x - 5| = 25$

b) $(x + 1)^2 - x(x + 3) = 4$

4. In the figure to the right, ABCD is a rectangle, and AE = DE. Prove that then BF = CG



5.. The following list shows some numbers, written by words in the language of a Pacific island nation. Each next number is equal to the previous one plus 2. Can you determine what these numbers are?

thabung ke nua lo
thabung ke nua vak
libenyita ke nua khasa
libenyita ke nua kun
libenyita ke nua thabung
libenyita ke nua thabung ke nua lo
libenyita ke nua thabung ke nua vak