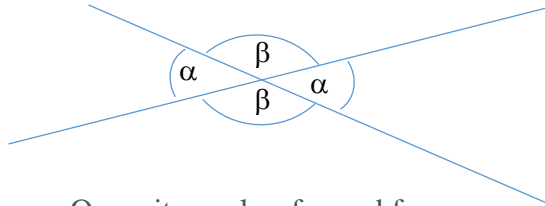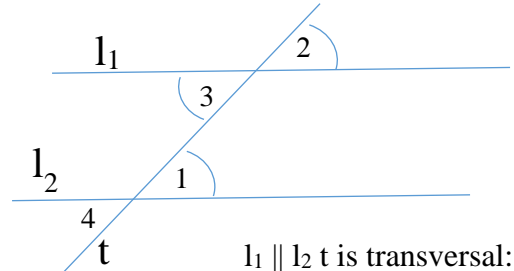Math 5B: Classwork 20
Homework #20 is due March, 8

# *Geometry: parallel lines, parallelogram*

Opposite angles, formed from
crossing straight lines, are equal.

$l_1 \parallel l_2$ t is transversal:
$$\angle 1 = \angle 2 = \angle 3$$

$\angle\, \alpha = \angle\alpha$ – opposite

$\angle\alpha + \angle\beta = 180^{\,0}$ – on a straight line,

Or complementary angles

$\angle 1 = \angle 3$ = alternate interior angles

$\angle 1 = \angle 2$ = corresponding angles

$\angle 4 = \angle 2$ = alternate exterior angles

**Theorem 1:** If two parallel lines (l1 and l2) are intersected by a third line (t), then the formed alternate interior angles are equal.

**Theorem 2:** If two alternate interior angles formed when two lines (l1 and l1) crossed by a third (t) are equal ($\angle 1 = \angle 3$), then the two lines (l1 and l2) are parallel.

**Theorem 3**: Show that the sum of angles in a triangle is always 180º.
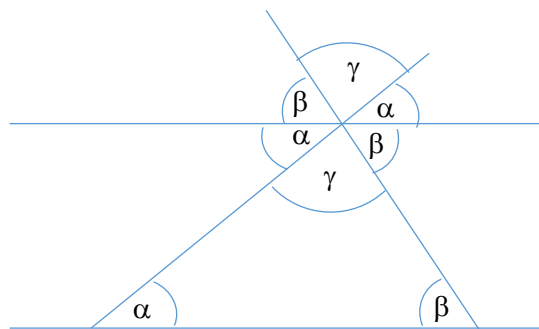
# Congruence

In general, two figures are called congruent if they have the same shape and size. We use the symbol $\cong$ to denote congruent figures: to say that $M_1$ is congruent to $M_2$, one writes $M_1 \cong M_2$.

The precise definition of what the "same shape and size" means depends on the figure. Most importantly, for triangles it means that corresponding sides are equal and corresponding angles are equal: $\Delta ABC \cong \Delta A'B'C'$ is the same as:

$$AB = A'B', \quad BC = B'C', \quad AC = A'C',$$

$$\angle A = \angle A', \quad \angle B = \angle B', \quad \angle C = \angle C'.$$

Note that for triangles, the notation $\Delta ABC \cong \Delta A'B'C'$ not only tells that these two triangles are congruent, but also shows which vertex of the first triangle corresponds to which vertex of the second one. For example, $\Delta ABC \cong \Delta PQR$ is not the same as $\Delta ABC \cong \Delta QPR$.

# Congruence tests for triangles

By definition, to check that two triangles are congruent, we need to check that corresponding angles are equal and corresponding sides are equal; thus, we need to check 6 equalities. However, it turns out that in fact, we can do with fewer checks.

***Rule 1 (Side-Side-Side rule).*** If $AB \cong A'B'$, $BC \cong B'C'$ and $AC \cong A'C'$ then $\Delta ABC \cong \Delta A'B'C'$.

This rule is commonly referred to as the ***SSS*** rule.

One can also try other ways to define a triangle by three pieces of information, such as two sides and an angle between them. We will discuss it next time. This rule – and congruent triangles in general – are very useful for proving various properties of geometric figures. As an illustration, we prove the following useful result.

***Theorem.*** Let $ABCD$ be a quadrilateral in which opposite sides are equal: $AB = CD$, $AD = BC$. Then $ABCD$ is a parallelogram.

Proof. Let us draw a diagonal $BD$. Then triangles $\Delta ABD$ and $\Delta CDB$ are congruent by ***SSS***; thus, the two angles labeled by letter $a$ in the figure are equal; also, the two angles labeled by letter $b$ are also equal. Thus, lines $BC$ and $AD$ are parallel (alternate interior angles!). In the same way we can show that lines $AB$ and $CD$ are parallel. Thus, $ABCD$ is a parallelogram. (It is also true in the opposite direction: in a parallelogram, opposite sides are equal. We will prove it next time.)
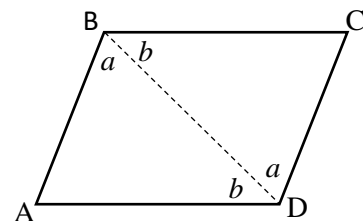
# *Homework*

1. An *n-gon* is called *regular* if all sides are equal
   and all angles are also equal.
   (a) How large is each angle in a regular
       hexagon (6-*gon*)?
   (b) Show that in a regular hexagon, opposite
       sides are parallel. (This is the reason why
       this shape is used for nuts and bolts).
       [Hint: show that each of the angles labeled
       by the letter *a* in the figure is equal to $60^\circ$,
       and then use the theorem about alternate
       interior angles.]
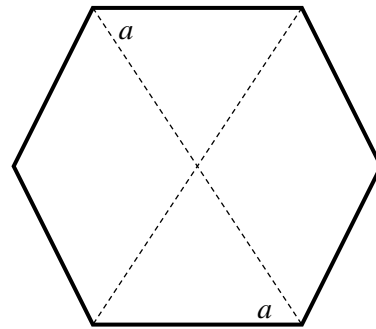
2. Let $ABC$ be a triangle in which two sides are
   equal: $AB = BC$ (isosceles triangle). We proved
   in class that if $M$ is the midpoint of the side $AB$,
   i.e. $AM = MB$, then
   - ✓ triangles $AMC$ and $BMC$ are congruent.
   - ✓ angles $A$ and $B$ are equal
   - ✓ angle $AMC = 90$ degrees

   So, in an isosceles the median is also a height.

   a) Please, review your notes and prove the
      above 3 points again!

   b) Can you prove the following: If in the triangle $\triangle ABC$ $\angle A = \angle B$ and the point $P$ of the
      side $AB$ is such that $CP$ is a height (angle $APC = 90^\circ$) then this triangle is isosceles?
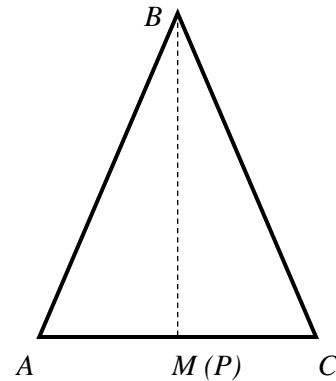
3. Simplify the expressions:
   (a) $x - (1 + 5x) =$
   (b) $2x - (3x^2 + x - 1) + (2 + 2x - x^2) =$
   (c) $3x(-2xy) =$
   (d) $(y - 5)(y - 1) - (y + 2)(y - 3) =$
   (e) $3(x - 1)^2 - 3x(x - 5) =$