

Homework 18: Review

1. Solve the following equations for *(don't forget to check your solutions)*:

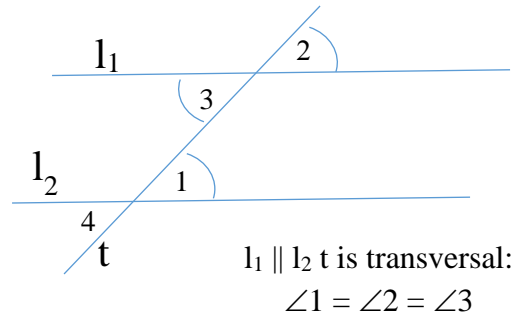
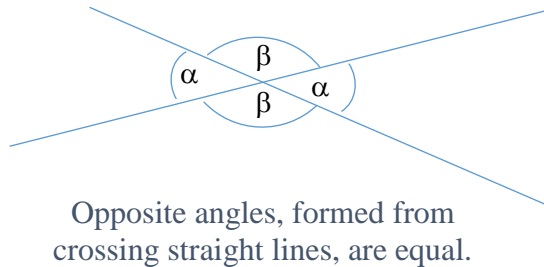
(a) $\frac{28x-14}{20-10x} + 5x = 4$

(b) $\frac{10y+30a}{60a-20y} - 6 = -1$, find y

2. Solve the equation

$$(9 - 2x)^2 - (2x - 7)(2x + 7) = 202$$

Geometry: parallel lines, parallelogram



$\angle \alpha = \angle \alpha$ – opposite

$\angle \alpha + \angle \beta = 180^\circ$ – on a straight line,

Or complementary angles

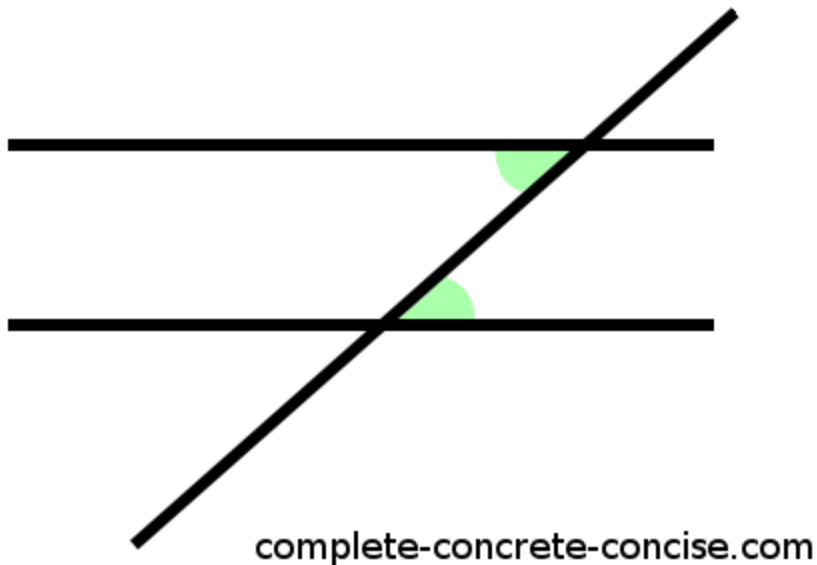
$\angle 1 = \angle 3$ = alternate interior angles

$\angle 1 = \angle 2$ = corresponding angles

$\angle 4 = \angle 2$ = alternate exterior angles

Theorem 1: If two parallel lines (l_1 and l_2) are intersected by a third line (t), then the formed alternate interior angles are equal.

This is illustrated in the image below:



We see two parallel lines and a third line (transversal) intersecting (crossing or cutting through) both of them. The green shaded angles are: (1) inside (between) the two parallel lines, (2) congruent (identical or the same), and (3) on opposite sides of the transversal.

This is true for the other two unshaded interior angles. It is also true for the alternate exterior angles (but not proved here).

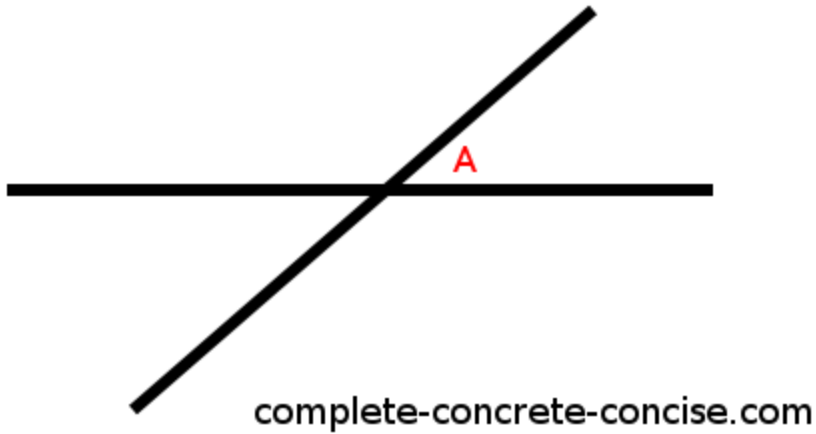
Axioms

Proofs are built on two things: (1) postulates, axioms, or hypotheses – these are things that are assumed to be true, but can't be proved. In general, we try to use the fewest number of axioms we can; (2) other proofs.

This proof depends on two axioms: (1) if you pick any two distinct points on a straight line, the angle between those two points will be 180° ; (2) if you take any two intersecting straight lines and shift one of the lines so it is in a different position, but still parallel to its original position, the angle between the two intersecting lines stays the same.

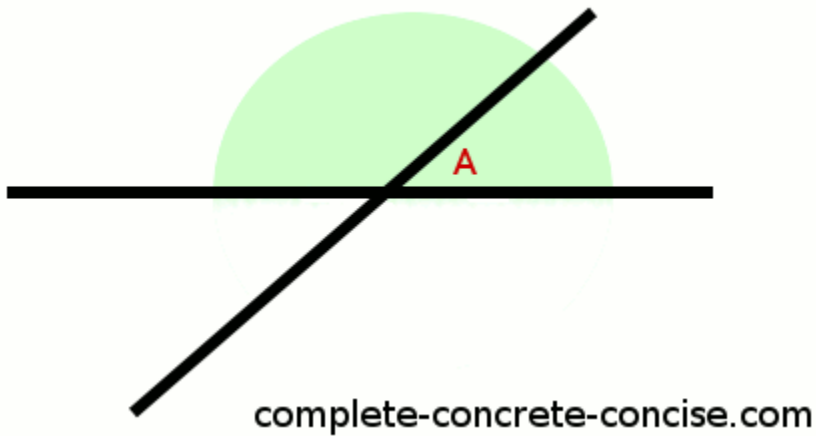
Proof

1) Consider two intersecting lines:

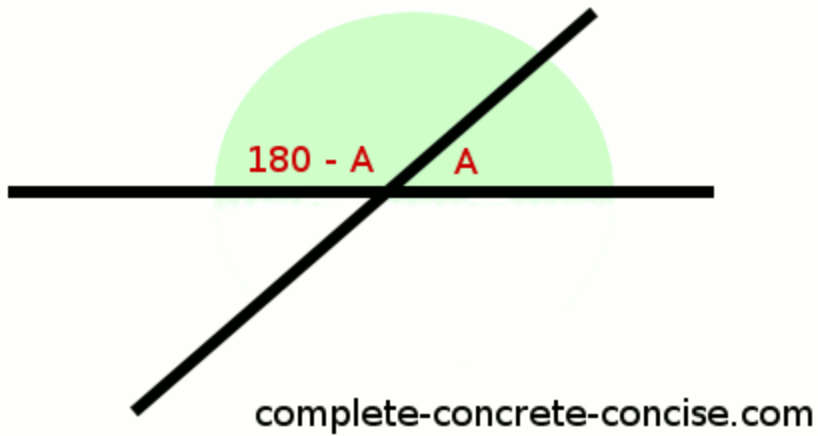


At the point of intersection, there is an angle (called A).

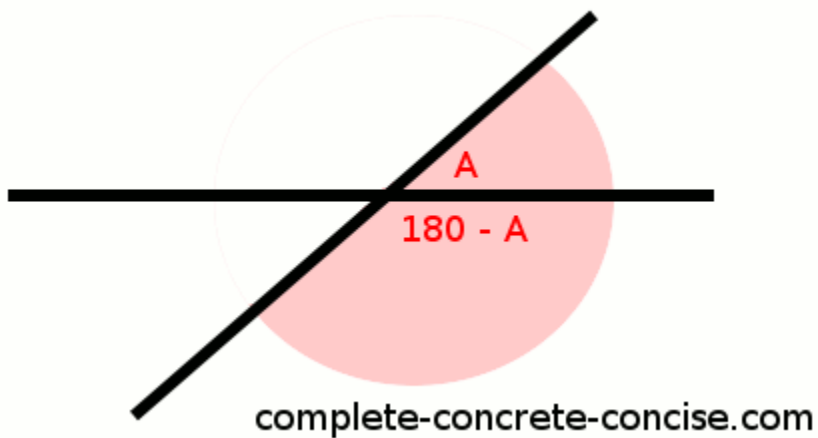
2) If the line is straight, the angle between any two points on that line must be 180° (axiom #1 from above):



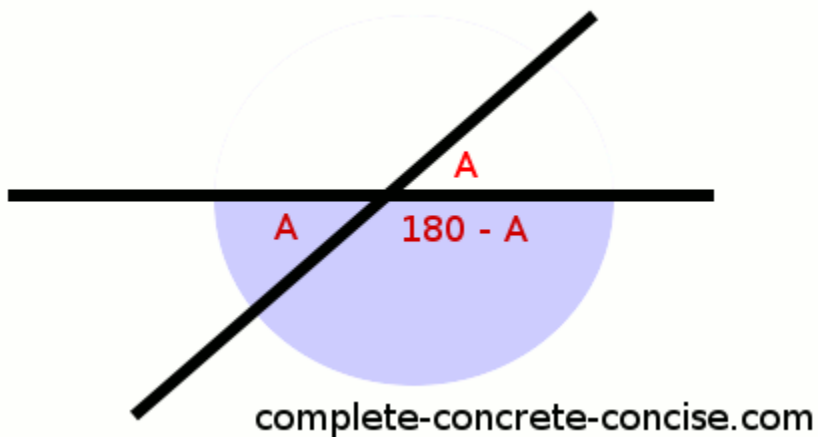
3) If one angle is A degrees, then obviously the other angle must be $180^\circ - A$:



4) This is true for both straight lines:



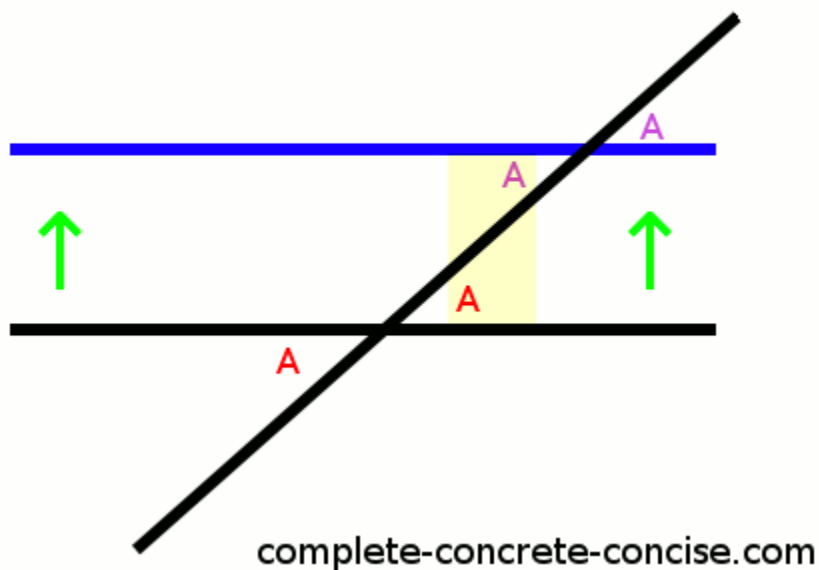
5) This means the remaining angle must be $180^\circ - (180^\circ - A) = A$ degrees:



This proves that angles on alternate sides of the transversal (at the point of intersection) are congruent (identical).

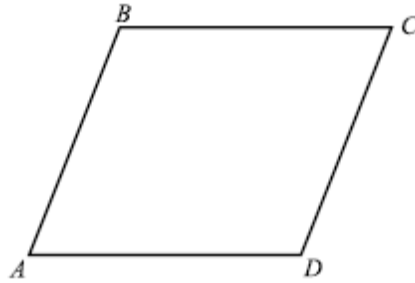
6) If we draw a line parallel to one of the lines (it doesn't matter which) and it intersects the other line (that line is now called a transversal), we know that the angles of intersection must be the same (axiom #2 from above).

If the angles of intersection are the same for both lines, then the alternate interior angles must be the same:



Theorem 2: If two alternate interior angles formed when two lines (l_1 and l_2) crossed by a third (t) are equal ($\angle 1 = \angle 3$), then the two lines (l_1 and l_2) are parallel.

Parallelogram: A parallelogram is a quadrilateral in which opposite sides are parallel. Parallelograms have a number of interesting properties, which we will study later.

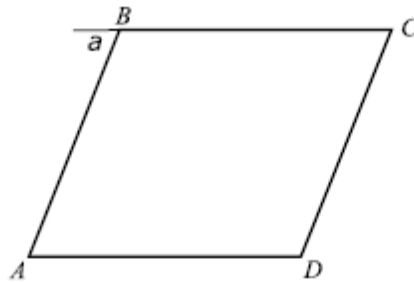


Sum of angles of an n-gon

Recall that sum of angles of a triangle is 180^0 . Since a quadrilateral can be cut into 2 triangles, sum of angles of a quadrilateral is $2 \times 180^0 = 360^0$. Similarly, for a pentagon we get 3×180^0 , and for an n-gon, the sum of angles is $(n - 2) \times 180^0$.

Classwork / Homework

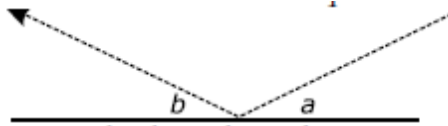
1. Is it true that any rectangle is also a parallelogram? Is it true that any parallelogram is a rectangle? Try to argue as carefully as you can.
2. Show that in a parallelogram, diagonally opposite angles are equal $\angle A = \angle C$, $\angle B = \angle D$ [Hint: see figure below, we did this in class, try to repeat the arguments]



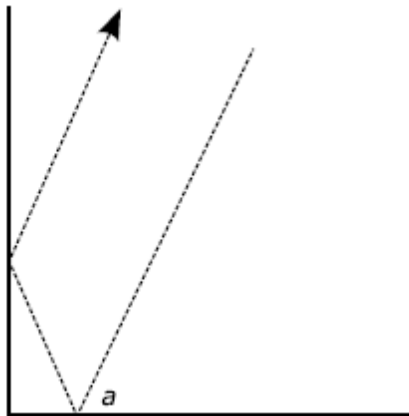
3. Show that the previous problem also works in the other direction: if in a quadrilateral, diagonally opposite angles are equal: $\angle A = \angle C$, $\angle B = \angle D$, then the quadrilateral must be a parallelogram.
4. Cut two identical paper triangles (the easiest way to do it is to fold a sheet of paper in two and then cut). Can you put these two triangles together so that they form a parallelogram? Will your method always work? Why?
5. Antonia and Gabi took a 9-mile trip in a rowboat. There was only one pair of oars, so they took turns rowing (however, they didn't time how long each of them was rowing, so

it could happen that one had rowed longer than the other). Antonia could row at the speed of 3 miles per hour; Gabi could only do 2 miles per hour. It took them 3.5 hours to complete the trip. Can you find out how long each of them was rowing?

6. An n -gon is called regular if all sides are equal and all angles are also equal.
 - (a) How large is each angle in a regular hexagon (6-gon)?
 - (b) How large is each angle in a regular heptagon (7-gon)?
7. The reflection law states that the angles formed by the incoming light ray and the reflected one with the surface of the mirror are equal: $\angle a = \angle b$



Using this law, show that a corner made of two perpendicular mirrors will reflect any light ray exactly back: the reflected ray is parallel to the incoming one.



[Hint: find the angle which each of these lines form with the horizontal]

This property – or rather, similar property of corners in three dimensions — is widely used: reflecting road signs, tail lights of a car, reflecting strips on clothing are all constructed out of many small reflecting corners so that they reflect the light of a car headlamp exactly back to the car.