

Math 5b: Classwork 15
Homework #15 is due January 27

Definition: A **rational** number is a number that can be in the form p/q where p and q are integers and q is not equal to zero. Example: $2/3$ is a rational number because 3 and 2 are both integers

Theorem: The square-root of 2 ($\sqrt{2}$) is not a rational number, i.e. it cannot be written as a fraction.

Proof: Let us assume that $\sqrt{2} = \frac{p}{q}$ where p and q are some whole numbers and the fraction $\frac{p}{q}$ cannot be simplified further. We can write:

$$(\sqrt{2})^2 = \left(\frac{p}{q}\right)^2$$

$$2 = \frac{p^2}{q^2}$$

So that:

$$2q^2 = p^2$$

Thus, p must be an even number and could be rewritten as: $p = 2m$. Substituting:

$$2q^2 = p^2 = 4m^2$$

So that:

$$q^2 = 2m^2$$

Thus, q must be an even number. This *contradicts* our initial assertion that $\frac{p}{q}$ could not be simplified further (at least each p , q could be reduced by one factor of 2 each). Therefore, we have proven by contradiction that $\sqrt{2}$ cannot be written as a rational number.

Homework

1. Simplify:

(a) $(\sqrt{17})^2$

(b) $(\sqrt{13})^4$

(c) $(\sqrt{11})^3$

(d) $(\sqrt{3^4 3^6})$

(e) $(\sqrt{2^4 3^5})$

2. Can one cut a square with the side of 1m from the circle of diameter 1.4m?

3. The side of an equilateral triangle is 1m. Find its height and area. Reminder: an equilateral triangle has all sides the same length.

4. Take a positive number $x < 100$ and using a calculator (or computer) calculate the number $\frac{x}{2} + \frac{1}{x}$. Call the result x and repeat the same calculation with the new x . Do it 10 times. Then take the result and square it. What did you get? Try to do the same thing starting with a different number. Is it surprising?
5. How many behemoths can one truck carry with a maximum load of 5 tonnes (5000 kg) if the weight of each behemoth is 1500 kg? How many crocodiles can the same truck carry if the weight of each crocodile is 175 kg?
6. Calculate:

$$(a) \quad (2^{-1})^2 \qquad (3^{-2})^{-2} \qquad ((-2)^{-1})^2 \qquad \left(\left(3\frac{1}{7}\right)^0\right)^{-6}$$

$$(b) \quad \left(\frac{2(a+1)^3(a+1)^4}{3(a+1)^3}\right)^{-1}$$

$$(c) \quad 2a^{-1} + 8(2a)^{-1} - 4\left(\frac{a^5}{a^4}\right)^{-1}$$

7.

Simplify the following expressions:

(a)

$$\frac{1}{x+1} - \frac{1}{x-1}$$

(b)

$$\left(1 + \frac{1}{x}\right) \div (x+1)$$

(c)

$$\left(1 + \frac{1}{x}\right) \div \left(1 - \frac{1}{x}\right)$$

8. Base 16 numbers:

a) add two base 16 numbers together:

$$\begin{array}{r} ABCD \\ + F23E \\ \hline \end{array}$$

$$\begin{array}{r} 3FBC \\ + A9F8 \\ \hline \end{array}$$

b) subtract two base 16 numbers :

$$\begin{array}{r} FDCB \\ - 9ABC \\ \hline \end{array}$$

$$\begin{array}{r} F35D \\ - 9C8A \\ \hline \end{array}$$

9* Problem we discussed in class. On the table in front of Tony there are 1000 quarters, 990 tails and 10 heads. Tony is blindfolded and cannot tell the difference between a head and a tail. Help Tony to split the coins in 2 groups, so that the number of heads in each group is the same.