Math 5b: Classwork & Homework 13
Homework #13 is due January 12.

Review

Powers: $a^n = a \times a \times a \times ... \times a$ (*n* times)

$a^0 = 1$ read: *a*-to-the-zero
$a^1 = a$ is just itself '*a*'
$(ab)^n = a^n \times b^n$
$a^n a^m = a^{n+m}$
$\frac{a^n}{a^m} = a^{n-m}$
$a^n = \frac{1}{a^{-n}}$, $a^{-n} = \frac{1}{a^n}$

Difference of squares formula: $(x - a)(x + a) = (x^2 - a^2)$
Square of the difference formula: $(a - b)(a - b) = (a - b)^2 = a^2 - 2ab + b^2$
Square of the sum formula: $(a + b)(a + b) = (a + b)^2 = a^2 + 2ab + b^2$

Binary Numbers: Numbers represented by using only 0s and 1s.
Powers of 2

n	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*
2^n	***1***	***2***	***4***	***8***	***16***	***32***	***64***	***128***	***256***	***512***

Example: Numbers in decimal notation can be presented like this (same as converting a number to a decimal notation): $351 = 3 \times 100 + 5 \times 10 + 1 \times 1$
Similarly, to convert a number into a binary, we need to represent it in powers of 2:

351 = **256** + 95 = **256 + 64** + 31 **= 256 + 64 + 16** + 15 **= 256 + 64 + 16 + 8** + 7 = **256 + 64 + 16 + 8 + 4 + 2 + 1**

$351 = 1\times2^8 + 0\times2^7 + 1\times2^6 + 0\times2^5 + 1\times2^4 + 1\times2^3 + 1\times2^2 + 1\times2^1 + 1\times2^0 = 101011111$

To convert number from binary to decimal we use the familiar rule where we multiply each digit by the position value in base 2. For example:

$1010 = 1\times2^3 + 0\times2^2 + 1\times2^1 + 0\times2^0 = 8 + 0 + 2 + 0 = 10$

Square-Root

The square-root of *a* is a number whose square is equal to *a*. For example: the square-root of 25 is 5 because $5^2 = 25$. Notation: square-root of a number, *a*, is commonly denoted as $\sqrt{a}$.
Similarly tob^n $(ab)^n = a^n b^n$, $\sqrt{ab} = \sqrt{a}\sqrt{b}$.
For example, $\sqrt{36} = \sqrt{9 \times 4} = \sqrt{9} \times \sqrt{4} = 3 \times 2 = 6$. And we also know that $\sqrt{36} = 6$.

Homework

1. Solve the following equations:
   (a) $2(x-1) = \frac{2}{3}(x+5)$

   (b) $2x - 5(x-7) = -1$

   (c) $\frac{x-2}{x-5} = 3$

   (d) $\frac{x-2}{x-5} + 5 = 3$

2. Do the following arithmetic operations with binary numbers. Do them without converting the numbers to decimal form:

   (a) 110101**b** + 111011**b**

   (b) 10101**b** × 1011**b**

   (c) (10101**b** + 1101**b**) × 10110**b**

3. Base 4 numbers:

   add two base 4 numbers together:

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  123          3201
+ 321        + 2310
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   *[Do not add in base 10 and translate the result to base 4, try performing addition in base 4, **think base 4**]*

4. The following is the beginning of a computer file. Can you decode it (assuming it is written in the standard, Latin 1, encoding)?
   (a) First, convert to base-10 numbers
   (b) **(Optional)** then look up the corresponding letters in the ISO/IEC 8859-1 table online https://en.wikipedia.org/wiki/ISO/IEC_8859-1

   01010100 01101111 01110000 00100000 01110010 01100101 01100011 01110010 01100101 01110100 00001010

5. Write the following expressions as powers with an appropriate base

   (a) $\frac{1}{2\cdot2\cdot2\cdot2\cdot2\cdot2\cdot2} =$

(b) $1mm = ?\, m$

(c) $1dm = ?\, m$

(d) $1km = ?\, m$

(e) $\frac{a}{a \cdot a \cdot a \cdot a \cdot a \cdot a \cdot a \cdot a} =$

6. "Jack has at least a thousand books", said Maria. "No, he has less than a thousand", said Daniel. He certainly has at least one book", said Kathy. If it is known that only one of the statements is true, how many books does Jack have?

7. Find the following square-roots. If you cannot find the number exactly, at least say between which two whole numbers the answer is, e.g. between 5 and 6.
   (a) $\sqrt{16}$
   (b) $\sqrt{81}$
   (c) $\sqrt{10{,}000}$
   (d) $\sqrt{10^8}$
   (e) $\sqrt{50}$

8. Find (Hint, you do not need to compute the number under the $\sqrt{\ }$)
   (a) $\sqrt{2^6 \times 7^2}$
   (b) $\sqrt{\frac{1}{16}}$
   (c) $\sqrt{\frac{4}{9}}$

9. If $a = 3^{-23}5^{17}$, $b = 2^{35}3^{-43}$, $c = 2^{47}5^{-18}$, and $d = 10^{19}2^{23}3^{-67}$ what is the value of *ab*? of *a/b*? abc? ab/c? abcd? ab/cd?

10. Anna has 60 coins which should be identical but one of them is fake. The fake one looks the same as all other coins but is lighter. Using balance scales, but not weights so you should put coins on both platforms, what is the fastest way to finding the fake coin? What would you do if you do not know whether the fake coin is lighter or heavier than the real ones?

11. A fish head weighs as much as the tail and half of the body together. The body weighs as much as the head and tail together. If the tail weighs 1 kg, how heavy is the fish?

12. *Here are phrases in Swahili with their English translations:

atakupenda – He will love you.
nitawapiga – I will beat them.
atatupenda – He will love us.
anakupiga – He beats you.
nitampenda – I will love him.
unawasumbua – You annoy them.

Translate the following into Swahili:

You will love them.
I annoy him.