Math 5b: Classwork & Homework 12
Homework #12 is due January 5.

Review

Powers: $\qquad\qquad\qquad\quad a^n = a \times a \times a \times ... \times a$ (*n* times)

$$a^0 = 1 \qquad\qquad \text{read: } a\text{-to-the-zero}$$
$$a^1 = a \qquad\qquad \text{is just itself `}a\text{'}$$
$$(ab)^n = a^n \times b^n$$
$$a^n a^m = a^{n+m}$$
$$\frac{a^n}{a^m} = a^{n-m}$$
$$a^n = \frac{1}{a^{-n}} \quad , \quad a^{-n} = \frac{1}{a^n}$$

1. If $a = 2^{19}3^{-91}$ and $b = 2^{-46}3^{-68}$ what is the value of *ab*? of *a/b*?

<u>**Difference of squares formula**</u>: $\qquad (x - a)(x + a) = (x^2 - a^2)$
<u>**Square of the difference formula**</u>: $\quad (a - b)(a - b) = (a - b)^2 = a^2 - 2ab + b^2$
<u>**Square of the sum formula**</u>: $\qquad (a + b)(a + b) = (a + b)^2 = a^2 + 2ab + b^2$

<u>**Binary Numbers:**</u> Numbers represented by using only 0s and 1s.
<u>***Powers of 2***</u>

n	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*
2^n	*1*	*2*	*4*	*8*	*16*	*32*	*64*	*128*	*256*	*512*

Example: Numbers in decimal notation can be presented like this (same as converting a number to a decimal notation): $351 = 3 \times 100 + 5 \times 10 + 1 \times 1$
Similarly, to convert a number into a binary, we need to represent it in powers of 2:
$\qquad 351 = \mathbf{256} + 95 = \mathbf{256 + 64} + 31 = \mathbf{256 + 64 + 16} + 15 = \mathbf{256 + 64 + 16 + 8} + 7 = \mathbf{256 +}$
$\qquad\qquad \mathbf{64 + 16 + 8 + 4 + 2 + 1}$
$\qquad 351 = 1{\times}2^8 + 0{\times}2^7 + 1{\times}2^6 + 0{\times}2^5 + 1{\times}2^4 + 1{\times}2^3 + 1{\times}2^2 + 1{\times}2^1 + 1{\times}2^0 = 101011111$

To convert number from binary to decimal we use the familiar rule where we multiply each digit by the position value in base 2. For example:
$\qquad 1010 = 1{\times}2^3 + 0{\times}2^2 + 1{\times}2^1 + 0{\times}2^0 = 8 + 0 + 2 + 0 = 10$

Classwork

1. Simplify the following and show the answer in the exponent (power) form

(a) $\dfrac{3^7 \cdot 2^7}{2^3 \cdot 2^4} =$

(b) $\dfrac{6^5 \cdot 2^4}{3^5 \cdot 2^2} =$

(c) $\dfrac{7^9 \cdot 2^5}{7^2 \cdot 2^4} =$

(d) $\dfrac{11^4}{11^2 \cdot 5^2 \cdot 5^3} =$

2. Solve the equations:

(a) $\dfrac{9}{13} z = 3$

(b) $|x| = 2$

(c) $|x + 11| = 2$

(d) $\dfrac{x}{2} + 1 = \dfrac{4x}{7}$

(e) $x = \dfrac{1}{4}x + 6$

(f) $\dfrac{x+3}{x+1} = 4$

3. Convert the decimal numbers to binary:
   9, 12

4. Convert the following binary numbers to decimal:
   101, 1001

Homework 12 problems

1. Solve the following equations:
   a) $3(x - 1) - 4 = 3x + 8$
   b) $\dfrac{1}{2}(x - 1) = -19$
   c) $|2x| = 10$

2. Simplify the following and show the answer in the exponent (power) form

(e) $7^4 \cdot 11^2 \cdot 11^{-5} \cdot 7^2 =$

(f) $\dfrac{3^{-5} \cdot 2^7}{3^{-3} \cdot 2^4} =$

(g) $\dfrac{42^2}{6^2} =$

(h) $\dfrac{3^5 \cdot 3^{-5}}{3^9} =$

(i) $\dfrac{x^2 \cdot y^2 \cdot x^{-3}}{x^2}$

3. Convert the decimal numbers to binary:
   9, 12, 24, 38, 45

4. Convert the following binary numbers to decimal:
   101, 1001, 10110, 11011, 10101

7. For the following problem, you need to know that the speed of light is about 300,000 km/sec, and one year is about 3×10^7 seconds.
   a) In astronomy, a common unit of distance is a light year: the distance light covers in one year. How many kilometers is it?
   b) Another common unit of distance in astronomy is a parsec, which is approximately equal to 3×10^{13} km.
   - Can you compute how many parsecs are there in one light year?
   - How many parsecs is the distance between Earth and Sun (The distance is about 1.5×10^8 km)?
   - How many parsecs is the distance between Earth and the Andromeda Nebula (2,000,000,000,000,000,000,000 km)?

8. If $a = 3^{19} 5^{-91}$, $b = 2^{-46} 3^{-68}$, $c = 2^{54} 5^{-8}$, and $d = 10^7 2^{-4} 3^{-5}$ what is the value of ab? of a/b? abc? ab/c? $abcd$? ab/cd?

9. How many zeroes does the number $4^{15} 5^{26}$ end with?