Review: Power rules, Powers of 2

Power of a number: $a^n = a \times a \times a \times \ldots \times a$ (n times, n is a whole number)

Special cases:
$a^0 = 1$ read: a-to-the-zero
$a^1 = a$ is just itself 'a'
$a^2 = a \times a$ read: a-squared
$a^3 = a \times a \times a$ read: a-cubed

Tricky cases: $1^1 =?$ $1^2 =?$ $1^3 =?$
what about $1^0 =?$ $0^0 =?$

Properties:
$(ab)^n = ab \times ab \times ab \times \ldots \times ab$ (n times)
$(ab)^n = (a \times a \times a \times \ldots \times a) \times (b \times b \times b \times \ldots \times b)$ (n times)
$(ab)^n = a^n \times b^n$

Similarly:
$a^n a^m = (a \times a \times a \times \ldots) \times (a \times a \times a \times \ldots)$ (n and m times, respectively)
$a^n a^m = a \times a \times a \times \ldots \times a \times a$ ($n+m$ times)
$a^n a^m = a^{n+m}$
$\frac{a^n}{a^m} = a^n \div a^m = a^{n-m}$

Powers of 2:
$2^1 = 2$ $2^2 = 4$ $2^3 = 8$ $2^4 = 16$ $2^5 = 32$
$2^6 = 64$ $2^7 = 128$ $2^8 = 256$ $2^9 = 512$ $2^{10} = 1024$
Note: $2^{10} = 1,024 \approx 1,000 = 10^3$

Example 1: (Indian chess legend)
A king once played a chess match with Krishna himself (posing as a sage) and lost. His bet was one grain of rice on the first square, two grains on the second, four on the third, and twice as many on every next square. How many were there on the last (64th) square?

The answer: on 1st square there was $1 = 2^0$ grain
on 2nd square there were $2 = 2^1$ grains
on 3rd square there were $4 = 2^2$ grains
on 4th square there were $8 = 2^3$ grains
…..
on 64th square there were 2^{63} grains

How many grains total?

Answer:
square 1 2^0 $= 2^1 - 1 = 1$
square 1+2 $2^0 + 2^1$ $= 2^2 - 1 = 3$
square 1+2+3 $2^0 + 2^1 + 2^2$ $= 2^3 - 1 = 7$
…
square 1+2+3+...+64 $2^0 + 2^1 + 2^2 + \ldots + 2^{63} = 2^{64} - 1 =?$

How much would it weight (approximately!) in tons if one grain weights 0.1gram?

Answer: $\approx 2^{64} = 2^4 \cdot (2^{10})^6 \approx 16 \cdot (10^3)^6 = 16 \cdot 10^{18} = 16,000,000,000,000,000,000$ grains weight $1,600,000,000,000$ tons!

Example 2: In some problems, one has to divide by 2 instead of multiplying.
Problem: a friend thinks of a number between 1 and 100, and will only answer 'yes' or 'no' to your questions. How many questions do you need to guess the number?
Solution: the quickest way is to ask a question which cuts the number of possibilities in half (this is called *bisection*; it is the same way one look for a word in a dictionary). So the first question should be: 'Is the number larger than 50?' If the answer is 'yes' we know that the number is between 51 and 100; if it is 'no' then is it between 1 and 50. Either way there are only 50 possibilities left. The next question should again cut the number in half (e.g. is the number larger than 75?' After n guesses, the number of remaining possible numbers is not more than $100/2^n$, so the number of needed questions is 7 (i.e. smallest number is n such that:
$2^n = 2^7 = 128 > 100$)

Classwork & Homework

1. Solve the following equations:
   (a) $5(x-1) - 4 = 3x + 1$ (b) $\frac{2}{3}(x-2) =- 18$ (c) $|2x+1| = 7$
   (c) $-|3x - 7 + 8x| =- 15$ (d) $\frac{x-8}{11} =- 35$
   (e) $\frac{x+16}{x} =- 7$ (f) $\frac{x}{x-7} = 5$
   (g) $\frac{x-6}{x-9} = 8$ (h) $\frac{x-15}{11-x} =- 12$
2. Simplify the expressions:
   (a) $(2z^2 \cdot 3z^3 \cdot z)^2$ (b) $(4c^2 \cdot c^3)^3$
   (c) $\left(\frac{5g^4b^5}{4g^2b^3}\right)^3$ (d) $\left(\frac{8dg^2}{3d^3g^4}\right)^3$
2. When Dennis was 27, his son was three years old. Now his son's age is one-third of Dennis' age. How old is each of the now?
3. How much rice would the sage win in a chess game if he asked to put rice only on black squares?
4. Lotus flowers are growing in a lake. Every day each lotus plant divides into two plants, so the area its leaves cover is doubled. In 30 days the whole lake is covered with lotus leaves. When was exactly half of the lake covered by leaves?
5. There are 15 samples of water from various wells. It is known that exactly one of them contains a dangerous chemical. A lab can test for the chemical, but the analysis is time consuming and expensive. Can you find the sample containing the chemical using fewer than 15 tests? How many tests are needed? [Hint: take a droplet of water from each sample and test the new combined sample … use the strategy in 'example 2' above.]
6. Write as powers with base 3:
   (a) 243×3^3 (b) $3^7 \times 3^{11} \times (-81)$ (c) $-3^5(27 - 3^4)$
7. Find the prime factorization of 500 and 1215. Express as a multiplication of powers. Find the greatest common factor (GCF).