

MATH 5: Semester 2: Homework 12

JANUARY 5, 2020

Powers of 2

n	0	1	2	3	4	5	6	7	8	9
2 ⁿ	1	2	4	8	16	32	64	128	256	516

Yes/no answers

We discussed that if you need to guess a number by asking yes/no questions, you can do it by asking n questions provided that the number of possibilities is at most 2^n . if you need to guess a number between 1–1000, you can do it in 10 questions because **1000 <= 1024**.

The same applies to finding a fake coin among many, and to other similar problems.

Binary numbers

Numbers in decimal notation can be presented like this

$$351 = 3 \cdot 100 + 5 \cdot 10 + 1 \cdot 1$$

Or like this

$$351 = 256 + 95 = 256 + 64 + 31 = 256 + 64 + 16 + 15 = 256 + 64 + 16 + 8 + 7 = 256 + 64 + 16 + 8 + 4 + 2 + 1$$

$$351 = 1 \cdot 2^8 + 0 \cdot 2^7 + 1 \cdot 2^6 + 0 \cdot 2^5 + 1 \cdot 2^4 + 1 \cdot 2^3 + 1 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0 = 101011111b$$

1. Solve the following equations:

a. $3(x - 1) - 4 = 2x + 8$

b. $\frac{1}{2}(x - 2) = -19$

c. $|2x| = 10$

d. $|2x - 5| = 10$

2. Convert decimal numbers to binary:

9, 12, 24, 38, 45

3. Convert the following binary numbers to decimal:

101, 1001, 10110, 11011, 10101

4. You have scales (with two platforms), a 1-gram weight and a large bag of sugar. What would be the fastest way to measure exactly 8 grams of sugar? Exactly 128 grams? Exactly 100 grams?
5. What is the largest number that can be written as a 5-digit binary number?
6. Is it possible to encode every letter of English alphabet by a 4-digit binary number? You can choose any way you like — for example encoding A as 0000, B as 0001, or by any other method. Would it be possible if we used 5-digit binary numbers? *[It is suggested to think first, not to go ahead and try all possibilities]*