school nova

Algebra.

Review. Factorization.

In mathematics factorization is a decomposition of on object into a product of other objects, or representation of an object as a product of 2 or more objects, which called 'factors'. For example we can represent the expression $a \times b + a \times c$ as a product of a and expression $(b + c)$.

$$a \times b + a \times c = a \times (b + c)$$

Or in a numerical expression:

$$7 \times 5 + 7 \times 3 = 7 \times (5 + 3)$$

Or a number can be representing as product of two or more other numbers, for example:

$$40 = 4 \times 10, \quad 36 = 6 \times 6$$

Does any natural number can be represented as a product of 2 or more numbers besides 1 and itself? Natural numbers greater than 1 that has no positive divisors other than 1 and itself are called prime numbers.

Even numbers are the numbers divisible by 2 (they have 2 as a divisor), so they can be factorize as 2 times something else. Can an even number be a prime number? Is there any even prime number?

> **Prime factorization** or integer **factorization** of a number is the determination of the set of **prime** numbers which multiply together to give the original integer. It is also known as **prime** decomposition.

Prime factorization process:

Prime factors of 168 are $2, 2, 2, 3, 7$ and prime factors of 180 are $2, 2, 3, 3, 5,$
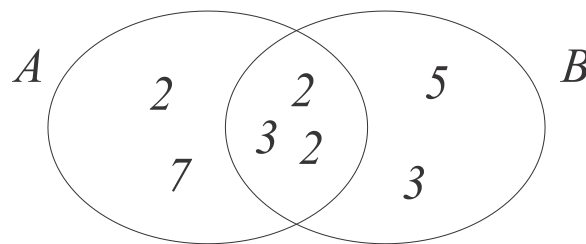
168	2
84	2
42	2
21	3
7	7
1	

180	2
90	2
45	3
15	3
5	5
1	

$$2 \times 2 \times 2 \times 3 \times 7 = 168; \quad 2 \times 2 \times 3 \times 3 \times 5 = 180$$

Any natural number has single unique prime factorization.

For Halloween the Jonson family bought 168 mini chocolate bars and 180 gummi worms. What is the largest number of kids between whom the Jonson can divide both kinds of candy evenly?

To solve this problem we have to find a number which can serve as a divisor for 168 as well as for 165. There are several such numbers. The first one is 2. Both piles of candy can be evenly divided between just 2 kids. 3 is also a divisor. The Jonson family wants to treat as many kids as possible with equal numbers of candy. To do this they have to find the Greatest Common Divisor (GCD), the largest number that can be a divisor for both (168 and 180) amounts of candy. Let's take a look at a set of all prime factors of 168 and 180. For 168 this set contains 2, 2, 2, 3, and 7. Any of these numbers as well as any of their products cab be divider for 168. The same goes for the set of prime factors of 180, which are 2, 2, 3, 3, and 5. It is easy to see that these two sets have common elements. It means that both numbers are divisible by any of these common elements and any their products. The largest product is the product of all common elements. This largest product is GCD.

A 2   2   5 B
    3   2
   7   3

$$168 = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 7$$
$$180 = 2 \cdot 2 \cdot 3 \cdot 3 \cdot 5$$

$$168 \div 12 = 14$$

$$180 \div 12 = 15$$

Between 12 kids they can divide both kinds of candy evenly.

A grasshopper jumps a distance of 12 centimeters each jump. A little frog jumps a distance of 15 centimeters each jump. They start hopping at the same time from the same point 0 and jump along the big ruler. What is the closest point on the ruler at which they can meet?

There are places on the ruler that both of them can reach after some number of jumps. One of such places is, of course, 12×15 cm. A grasshopper can make 15 jumps while a little frog can make only 12 jumps. Will 12x15 be the only place where they can meet or there are some other places? If this is the case, we have to find a number that is divisible by both 12 and 15. Take into account that 12×15, as well as any product of 12×15 is divisible by both 12 and 15.
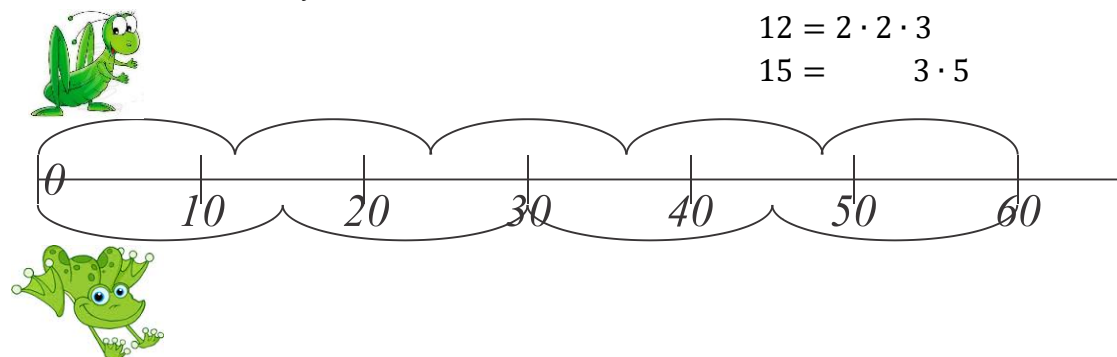
$$12 = 2 \cdot 2 \cdot 3$$
$$15 = \quad\; 3 \cdot 5$$
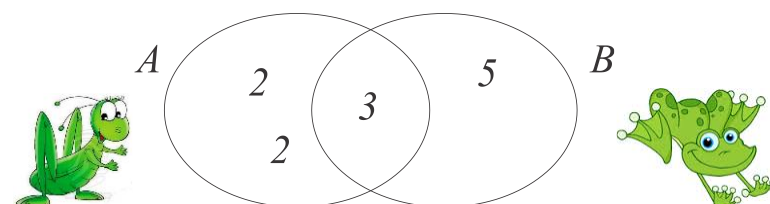

Is there are any number which is less then 12×15 and still divisible by 12 and 15? Prime factorization of 12 and 15:

12	2	15	3
6	2	5	5
3	3	1	
1			

$$12 \times 15 = (2 \times 2 \times 3) \times (3 \times 5)$$

The number which we are looking for has to be a product of prime factors of either 12 or 15, so it should be a union of two sets – set of prime factors of 12 and 15.

$$2 \times 2 \times 3 \times 5 = 60$$

60 is the smallest number, which is divisible by 12 and 15, LCM.

Eratosthenes proposed a simple algorithm for finding prime numbers. This algorithm is known in

mathematics as the Sieve of Eratosthenes.

In mathematics, the sieve of Eratosthenes, one of a number of prime number sieves, is a simple, ancient algorithm for finding all prime numbers up to any given limit. It does so by iteratively marking as composite, *i.e.*, not prime, the multiples of each prime, starting with the multiples of 2.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Exercises.

1. Two buses leave from the same bus station following two different routes. For the first one it takes 48 minutes to complete the roundtrip route. For the second one it takes 1 hour and 12 minutes to complete the round-trip route. How much time will it take for the buses to meet at the bus station for the first time after the have departed for their routes at the same time?

2. A florist has 36 roses, 90 lilies, and 60 daisies. What is largest amount of bouquets he can create from these flowers evenly dividing each kind of flowers between them?

3. Mary has a rectangular backyard with sides of 48 and 40 yards. She wants to create square flower beds, all of equal size, and plant different kind of flowers in each flower bed. What is the largest possible size of her square flower bed?

4. Factorize:
$15a + 15b =$
$10x + 2y =$
$36w - 6 =$
$100 - 25x =$

5. Find all prime factors of the following numbers:
66, 28, 128, 555, 1233

6. Robert's step length is 50 cm, and his father's length is 70 cm. Robert claims that for the first time, after taking an integer number of steps, he and his father will be at the same distance from the beginning of the path after 3 meters, and father will not agree. Who is right in this dispute?

7. Find GCD (GCF) of
   a. 420 and 450,
   b. 810, 945 and 1125

8. Find LCM of
   a. 8 and 12
   b. 15, 18, and 21

9. Prove that for any two numbers a and b $\quad GCF(a, b) \cdot LCM(a, b) = a \cdot b$