Math 4d. Class work 24.

1. **Exponent.**

Exponentiation is a mathematical operation, written as $\boldsymbol{b^n}$, involving two numbers, the **base** *b* and the **exponent** *n*. When *n* is a positive integer, exponentiation corresponds to repeated multiplication of the base: that is, $\boldsymbol{b^n}$ is the product of multiplying *n* bases:

$$b^n = \underbrace{b \times \cdots \times b}_{n}$$

In that case, $\boldsymbol{b^n}$ is called the *n*-th power of *b*, or *b* raised to the power *n*.

Properties of natural exponent:

If the same base raised to the different power and then multiplied:

$$b^3 \cdot b^4 = (b \cdot b \cdot b) \cdot (b \cdot b \cdot b \cdot b) = b \cdot b \cdot b \cdot b \cdot b \cdot b \cdot b = b^7$$

Or in a more general way:

$$b^n \cdot b^m = b^{n+m}$$

If the base raised to the power of n then raised again to the power of m:

$$(b^2)^3 = (b \cdot b)^3 = (b \cdot b) \cdot (b \cdot b) \cdot (b \cdot b) = b^{2 \cdot 3}$$

$$(b^m)^n = b^{mn}$$

If we want to multiply $b^n = \underbrace{b \cdot b \cdot b \ldots \cdot b}_{n\ times}$ by another b we will get the following expression:

$$b^n \cdot b = \underbrace{b \cdot b \cdot b \ldots \cdot b}_{n\ times} \cdot b = \underbrace{b \cdot b \cdot b \cdot b \ldots \cdot b}_{n+1\ times} = b^{n+1} = b^n \cdot b^1$$

In order to have the set of power properties consistent, $b^1 = b$ for any number b.

If we multiply b^n by 1, we won't change anything, so we can write

$$b^n \cdot 1 = b^{n+0} = b^n \cdot b^0$$

In order to have the set of power properties consistent, $b^0 = 1$ for any number $b \neq 0$

If two different bases raised to the same power, then:

$$(a \cdot b)^3 = (a \cdot b) \cdot (a \cdot b) \cdot (a \cdot b) = a \cdot a \cdot a \cdot b \cdot b \cdot b = a^3 b^3$$

$$(a \cdot b)^n = a^n b^n$$

The exponent indicates how many copies of the base are multiplied together. For example, $3^5 = 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 = 243$. The base 3 appears 5 times in the repeated multiplication, because the exponent is 5. Here, 3 is the *base*, 5 is the *exponent*, and 243 is the *power* or, more specifically, *the fifth power of 3*, *3 raised to the fifth power*, or *3 to the power of 5*.

1.

$2^3 \cdot 2^2 =$ $\quad$ $(2^3)^2 =$

$5^2 \cdot 5 =$ $\quad$ $(3^7)^2 =$

$2^5 \cdot 2^3 \cdot 2 =$ $\quad$ $(n^5)^3 =$

2. Write the following expressions as a product or power:

a. $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$;
b. $2 + 2 + 2 + 2 + 2$;
c. $a \cdot a \cdot a$;
d. $a + a + a$;
e. $\underbrace{x \cdot x \cdot \ldots \cdot x}_{20\ times}$;
f. $\underbrace{x + x + \cdots + x}_{20\ times}$;

3. Write the following expressions in a shorter way:

Example: $7 \cdot 7 \cdot 7 \cdot 8 \cdot 8 \cdot 8 \cdot 8 \cdot 9 \cdot 9 \cdot 9 \cdot 9 \cdot 9 = 7^3 \cdot 8^4 \cdot 9^5$

a. $2 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 7 \cdot 7$;
b. $\underbrace{3 \cdot 3 \cdot \ldots \cdot 3}_{n\ times} \cdot \underbrace{5 \cdot 5 \cdot \ldots \cdot 5}_{m\ times}$

$$\underbrace{(-4) \cdot (-4) \cdot \ldots \cdot (-4)}_{k\ times} \cdot \underbrace{6 \cdot 6 \cdot \ldots \cdot 6}_{l\ times}$$

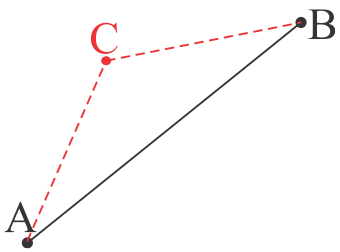
4. Compare the numbers:

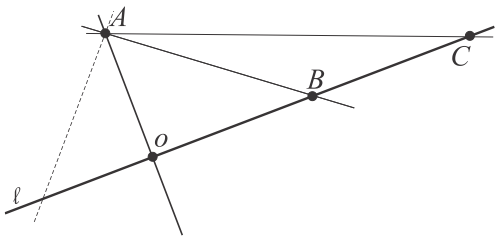
$a.\ 5^3 \quad 5 \cdot 3$ $\qquad$ $b.\ 12^2 \quad 12 \cdot 2$

$c.\ 2^5 \quad 5^2$ $\qquad$ $d.\ 3^4 \quad 4^3$

$e.\ 5^3 \quad 5 \cdot 3$ $\qquad$ $f.\ 2^4 \quad 4^2$

The shortest distance between two points is a part of a straight line passing through these two points (a segment). The distance between a point and a line is the distance between the point and the point of intersection of the line and the perpendicular drawn from the point to the line.





AO is a perpendicular drawn from the point A to the line. |AO| is the distance between the point A and the line l.

*On a picture on the right the caterpillar wants to go from vertex G to vertex E on the cube. Draw the shortest way for it to go. What will be the shortest way to go from the vertex G to vertex A? Find all possible solutions.