Math 4d. Class work 23.

There are 5 chairs and 5 kids in the room. In how many ways can kids sit on these

chairs? The first kid can choose any chair. The second kid can choose any of the 4 remaining chairs, the third child has a choice between the three chairs, and so on. Therefore, there are  $5 \times 4 \times 3 \times 2 \times 1$  ways how all of them can choose 1, can be written as 5!. By definition:



 $5 \times 4 \times 3 \times 2 \times 1 = 5!$  or  $n \times (n-1) \times (n-2) \times ... \times 3 \times 2 \times 1 = n!$ 

Write the following expressions as a factorial and vice versa:

Example:  $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 7!$ ,  $4! = 4 \times 3 \times 2 \times 1$ 

 $10 \times 9 \times 8 \times \dots \times 3 \times 2 \times 1 =$ 6! = $b \times (b-1) \times (b-2) \times ... \times 3 \times 2 \times 1 =$ c! =

1. Simplify the following fractions:

$$\frac{5!}{7!} = \frac{n!}{(n-2)!} =$$

- 2. How many different ways are there to put 64 books on the shelf?
- 3. There are 20 students in the 4<sup>th</sup> grade. They have to choose a president, and a vice president of the class. How many different ways are there to do it?





4. There are 20 students in the 4<sup>th</sup> grade. They have to choose a teem of two studets to go to the math competition. How many different ways are there to do it?

How many different 3-digit numbers can we create using 8 digits, 1, 2, 3, 4, 5, 6, 7, and 8 without repetition of the digits, i.e. such numbers that only contain different digits? How many different ways are there to choose a team of 3 students out of 8 to participate in the math Olympiad.

What are the similarities in these two problems? Can you see the difference between them?



In both cases, we have 8 possible ways to choose the first item (digit or student), 7 possible ways to choose the second item, and 6 different ways to choose the third one. So, there are  $8 \cdot 7 \cdot 6$  different 3-digit numbers created from digits 1, 2, 3, 4, 5, 6, 7, and 8 and  $8 \cdot 7 \cdot 6$  different teams of 3 students out of 8. Or not?



We can create numbers 123, 132, 213, 231, 321, 312 and they are all different numbers. If we chose Mike, Maria, and Jessika, a team of 3 students for the math Olympiad, it doesn't matter in which order we wrote their names.

In the first case, we have  $8 \cdot 7 \cdot 6$  ways to create a 3-digit number out of 8 digits. In the second case for each group of 3

kids we will count 6 times (3! – number of ways to put 3 kids in line) more possible choices than there really are. So the total number of the way to choose the teem is

$$\frac{8 \cdot 7 \cdot 6}{3 \cdot 2 \cdot 1}$$

5. In the restaurant, there are 3 choices of starters, 4 choices of entrees and 5 choices of tasty desserts in the fix price dinner menu. How many different ways are there to fix a dinner for the restaurant's clients?

- 6. How many two-digit numbers can be composed from digits 1, 2, 3 without repetition of digits?
- 7. How many two-digit numbers can be composed from digits 1, 2, 3, if repetition is allowed?
- 8. Peter took 5 exams at the end of the year. Grade for exams are A, B, C, D. How many different ways are there to fill his report card?
- 9. There are red and green pencils in a box. How many pencils do you have to take out of the box without seeing them to be sure that you have at least 2 pencils of the same color?
- 10. If there are pencils of 5 different colors in a box, how many pencils do you have to take out to be sure that you have at least 2 of the same color? 3 of the same color?
- 11. There are 10 pairs of red gloves and 10 pairs of black gloves in a box. How many gloves do you have to take out to be sure that you have a pair of gloves that you can wear?
- 12. Write the expression for the perimeter and area of the figure.

