Math 4B. Handout #22 Combinations

Combinations

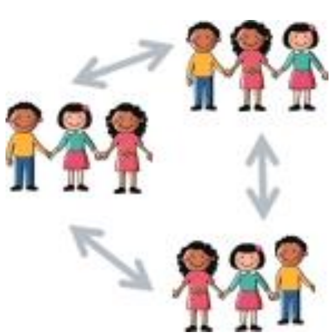
1) How many different 3-digit numbers can we create using 8 digits, 1, 2, 3, 4, 5, 6, 7, and 8 without repetition of the digits, i.e. such numbers that only contain different digits?

2) How many different ways are there to choose a team of 3 students out of 8 to participate in the math Olympiad.

In both cases, we have 8 possible ways to choose the first item (digit or student), 7 possible ways to choose the second item, and 6 different ways to choose the third one. So, there are $8 \cdot 7 \cdot 6$ different 3-digit numbers (permutations) created from digits 1, 2, 3, 4, 5, 6, 7 and 8.

$$P(8,3) = 8 \cdot 7 \cdot 6 = \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = \frac{8!}{5!} = \frac{8!}{(8-3)!}$$

How about the teams of 3 students? Is it $8 \cdot 7 \cdot 6$ different teams of 3 students out of 8?



We can create numbers 123, 132, 213, 231, 321, 312 and they are all different numbers (permutations -the number of ways to arrange three digits in a three-digit number 3!)

If we chose Peter, Maria, and Jessika, a team of 3 students for the math Olympiad, it doesn't matter in which order we pick their names. There is only 1 way to pick these 3 students.

So for each group of 3 kids we will count 6 times (3! – number of ways to put 3 kids in line) more possible choices than there really are. So in order to find those choices/combinations of kids we need to divide permutations by the number of ways we can arrange 3 kids in 3 places with is 3!.

$$8C3 = \frac{8 \cdot 7 \cdot 6}{3!} = \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{3! \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = \frac{8!}{3! \cdot 5!} = \frac{8!}{3! \cdot (8-3)!}$$

Combinations are the ways of choosing objects from a set of an objects and their order does NOT matter. To get from a **permutation** to a **combination**, we divide by the total number of ways to order the objects we choose.

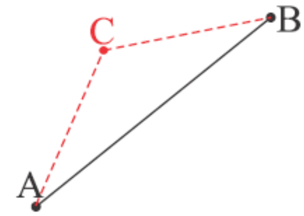
With **permutations** we care about the order of the elements, whereas with **combinations** we don't. For example, say your locker "code" is 5782. If you enter 2875 into your locker it won't open because it is a different ordering (aka **permutation**) all though the combination of numbers is the same.

We write permutation as ${}_{10}P_4$ which means: find all possible permutations (orders) of 4 objects if you have a choice of 10. In this case, find all possible locker codes. So: A "**combination** lock" should really be called a "**permutation** lock". The order you put the numbers in matters. (A true "**combination** lock" would accept both 5782 and 2875 as correct.)

We write combination as ${}_{10}C_4$ which means: Find all possible combinations of 4 objects if you have a choice of 10. Say you have 10 friends and you can invite only 4 of them to a movie, because you only have that many tickets to spare. What are the combinations? The order does not matter in this case.

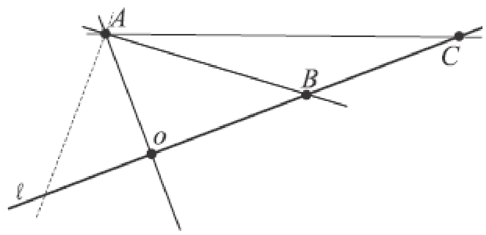
- Your odds of being struck by lightning this year are **1 in 960,000**.
- In your lifetime those odds drop to about 1 in 12,000.
- Your odds of being struck by lightning twice in your lifetime are 1 in 9 million
- The odds of getting attacked and killed by a shark are **1 in 3,748,067**.
- Odds of dying from fireworks (**1 in 340,733**) or drowning (**1 in 1,134**)

Geometry.



The shortest distance between two points is a part of a straight line passing through these two points (a segment).

The distance between a point and a line is defined as the shortest **distance between** a fixed **point** and any **point** on the **line**. It is the length **of** the **line** segment that is perpendicular **to** the **line** and passes through the **point**



AO is a perpendicular drawn from the point A to the line. |AO| is the distance between the point A and the line l**.**

The **distance between two parallel lines** in the length of perpendicular drawn from a point on one **line** to another.