Math 3, Classwork 17

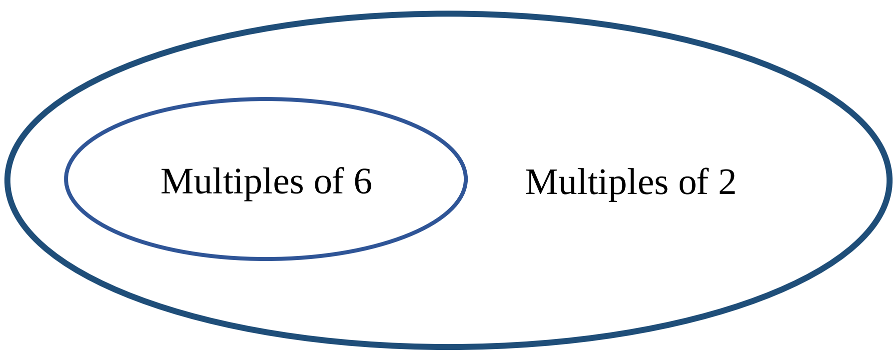
WARM UP

1. Multiplication Gym – 3 minutes

2. Ashton makes a sequence of five numbers. The 1st number is 2. The last number is 14. His rule is to add the same number each time. What are the missing numbers?

2, _____, ______, ______, 14.

3. Look at the Venn diagram below. Write the numbers 8, 9, 10, 11, 12 and 13 in the correct places on the diagram.



4. Write an algebraic expression for each statement.

a) Julia had N red apples, M yellow apples, K green apples, and 15 blue balloons. She shared the apples with her five friends equally. How many apples did each friend get? ______________________________

b) Each student in the class solved C multiplication problems and K word problems. If there were N children in the class, how many problems did they solve in total?

____

c) Milan had M dollars. How much money does he have left after he bought five books which cost X dollars each and N pens which costs 3 dollars each?

____

d) Elliot paid 15 dollars for N t-shirts. How much will he pay for M t-shirts?

____

LAWS OF ARITHMETIC – REVIEW

Addition and multiplication are both ***commutative***.

This means that $a + b = b + a$ and $a \times b = b \times a$ for every pair of numbers a and b

Commutativity means that we do not have to worry about whether we calculate $a + b$ or $b + a$ because the answer is the same.

Addition and multiplication are both ***associative***

This means that $a + (b + c) = (a + b) + c$ **and** $a \times (b \times c) = (a \times b) \times c$ for every pair of numbers a and b

Commutativity means that we do not have to worry about whether we calculate $a + b$ or $b + a$ because the answer is the same.

Associativity ensures that it makes no difference which of the two operations is calculated first in both cases: a + b + c and a × b × c

Commutativity and associativity are properties of a **single operation**.

The equation $3 \times (2 + 4) = (3 \times 2) + (3 \times 4)$ is an example of the **distributivity** of multiplication over addition. In general,

$a \times (b + c) = (a \times b) + (a \times c)$ for any numbers a, b and c

We can distribute multiplication over addition from the right, so

$(a + b) \times c = (a \times c) + (b \times c)$ for any numbers a, b and c

We can distribute multiplication over subtraction from both the left and the right, so

$a \times (b - c) = (a \times b) - (a \times c)$, and

$(a - b) \times c = (a \times c) - (b \times c)$ for any numbers a, b and c

All the above are called **distributive laws**.

When evaluating expressions, you don't always have to follow the order of operations strictly. Sometimes you can play around with the expression first. You can commute (with addition or multiplication), associate (with addition or multiplication), or distribute (multiplication or division over addition or subtraction). Know your options!

5. *Example*: $(32 \times 4) \times (25 \times 10) \times (10 \times 2) = 32 \times (4 \times 25) \times (10 \times 10) \times 2$

Why is the 2nd grouping is more convenient than the 1st?

6. Write the correct sign <, > or = to make these statements correct:

a) (8 + 5) – 7 … (8 + 7) – 5
b) 2 × (3 + 4) … (2 × 3) + 4
c) (10 × 5) ÷ 2 … 10 × (5 ÷ 2)

7.

Put the parenthesis to the following equalities to make them correct.

a) 6 + 2 × 5 = 40
b) 3 × 4 + 2 = 18
c) 3 + 4 × 2 + 4 = 42
d) 4 + 3 + 2 × 2 = 18

NEW MATERIAL I

What is the greatest number which can be placed in the parenthesis?

8. *Example: 20 × (   ) < 85. Think: how many groups of 20 are there in number 85? Answer: 4*

50 × (   ) < 156   70 × (   ) < 232   80 × (   ) < 438

(   ) × 20 < 108   (   ) × 30 < 149   (   ) × 40 < 278

9. Julia and Victoria decided to make a present for their grandparents – a photo album. They had 46 photographs to put in the album. A full-page holds six photographs. What is the smallest number of pages girls should use to put all the photos? How many photos will the last page hold?

Division with remainders. All numbers are whole numbers!

Division is different from addition, subtraction, and multiplication in that having a remainder is possible. A remainder is simply a portion left over from the division.

$$m \div n = q + r, m > n, r < q$$

In general, if *m* is **dividend**, *n* is **divisor**, *q* is **quotient** and *r* is **remainder** then $m = n \times q + r$. Divisor *n* should always be greater than remainder *r*.

Example: 10 ÷ 3 = 3r1   9 ÷ 3= 3r0   8 ÷ 3 = 2r2   7 ÷ 3= 3r1

Remainer from the division of any number by 3 is always equal either 0 or 1 or 2.

10. a) If Jonathan's age is divided by 2 or 3, there will be 1 left over. If his age is divided by 7, there is no remainder. How old is Jonathan? ________________

b) If Ronav's age is divided by 2 or 4, there will be 1 left over. If his age is divided by 3, there will be no remainder. How old is Ronav, if now he is younger than 10 years old? _______.

If $m < n$ (dividend is smaller than divisor), then quotient *q* equals 0 and the remainder is equal to the dividend.

$$m \div n = 0\ rm, m < n$$

Example: 5 ÷8 = 0 r5. Checking 8 × 0 + 5 = 5 and 5 < 8

11. Is it possible to get a remainder 5 or 6 or 7 while dividing a number by 4? _______

12. What are the remainders that can be obtained while dividing a number by 6? Assume that a dividend is greater than 6. Give examples for each remainder.

__________________ ______________ ________________

__________________ ______________ ________________

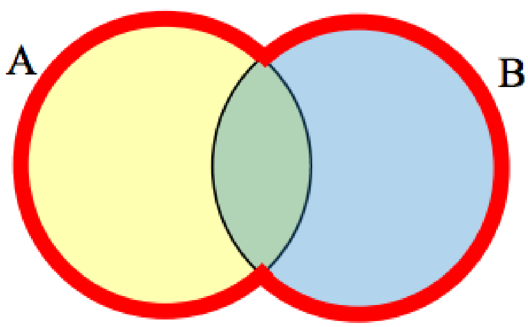
NEW MATERIAL II

The **union** of two sets contains all the elements contained in either set (or both sets). The union is notated $A \cup B$. More formally, $x \in A \cup B$ if $x \in A$ or $x \in B$ (or both)
The **intersection** of two sets contains only the elements that are in both sets. The intersection is notated $A \cap B$. More formally, $x \in A \cap B$ if $x \in A$ and $x \in B$.

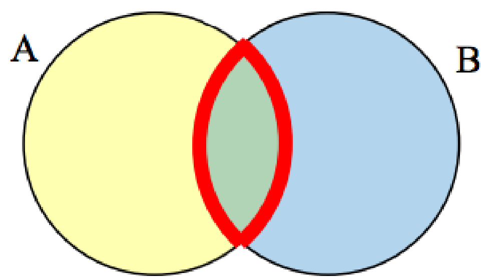
To visualize the interaction of sets, John Venn in 1880 thought to use overlapping circles, building on a similar idea used by Leonhard Euler in the eighteenth century. These illustrations now called **Venn Diagrams**.

A Venn diagram represents each set by a circle, usually drawn inside of a containing box representing the universal set. Overlapping areas indicate elements common to both sets.

A ∪ B contains all elements in either set (union)
$A \cap B$ contains only those elements in both sets—in the overlap of the circles.



$A \cup B$



$A \cap B$

13. *Example:* Consider the sets:

A = {red, green, blue}, B = {red, yellow, orange}, C = {red, orange, yellow, green, blue, purple}. Find the following:

a) Find $A \cup B$
b) Find $A \cap B$

Answers: a) The union contains all the elements in either set: $A \cup B$ = {red, green, blue, yellow, orange} Notice we only list red once.

b) The intersection contains all the elements in both sets: $A \cap B$ = {red}

14. There are 3 sets – H, F and W. Suppose H = {cat, dog, rabbit, mouse}, F = {dog, cow, duck, pig, rabbit}, and W = {duck, rabbit, deer, frog, mouse}.

1. Find a set which is $(H \cap F) \cup W$___________________________________
2. Find a set which is $H \cap (F \cup W)$ ___________________________________

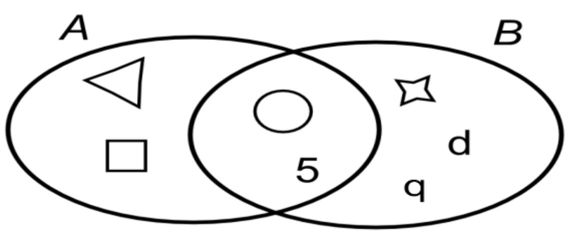
A **universal set** is a set that contains all the elements we are interested in. This would have to be defined by the context.

Example: If we were discussing searching for books, the universal set might be all the books in the library.

Universal set is usually represented by a box and all other sets are drawn inside this box

15. Consider sets **A** A = {◁ , □ , ○ , 5 } B = {○, ✧ , q , 5, d } **and B**:

By using { }, define the elements of the set **A∩B**. ____________________



16. A survey asks 200 people "What beverage do you drink in the morning", and offers choices:

- Tea only
- Coffee only
- Both coffee and tea

Suppose 20 report tea only, 80 report coffee only, 40 report both.

a) How many people drink tea in the morning? ____________________

b) How many people drink neither tea nor coffee? _________________

Create a Venn diagram to help you to solve a problem.

Did you know …

By Kent Haines.

At first, people invented math systems so they could count sheep, loaves of bread, and other tradable goods.

But as people counted and counted and counted, they realized they could make a shortcut. Instead of counting one group of sheep and then another group, why not just add the two groups together? And so **addition was born**. Of course, shortly thereafter people realized they needed an operation that undid addition, and so **subtraction was developed** as a related operation. For a long time, addition and subtraction were the only operations.

But after a while, people got sick of adding the same number over and over again. Eight people have how many fingers? Well 10 + 10 is 20 and then 20 + 10 is 30 and then 30 + 10 is 40...

So ancient mathematicians invented a beautiful shortcut for adding the same number repeatedly. They called it **multiplication**. And to undo multiplication, people invented **division**. Two more inverse operations.

Whenever people had a big long math problem, they would multiply and divide anywhere they could before moving to addition and subtraction. Multiplication and division were shortcuts, of course, so it made perfect sense to do them first.

And that's how it was for hundreds of years. People would multiply and divide, and then add and subtract. But over time, people got sick of repeatedly multiplying the same number over and over again.

So they invented yet another shortcut, known as the exponent. It's a quicker way of doing repeated multiplication, which made things much faster. And at the same time, people came up with its inverse, which is finding the root of a number.

At this point, people felt pretty good with their operations. You do exponents and roots first, then multiply and divide, and then add and subtract. First the shortest shortcut, then the next one, and finish up with old-fashioned adding and subtracting.

But what if you *want* to add first? How do you communicate that to other people? You need some sort of symbol that indicates a smaller, special group of numbers and operations. So people invented **parentheses, brackets,** fraction bars, and all sorts of other grouping symbols. So you do the math inside grouping symbols first, then do exponents and roots, then multiply and divide, and finally add and subtract.