MATH 10
ASSIGNMENT 6: ROUCHÉ-CAPELLI THEOREM

NOV 3, 2019

Today, the goal is to understand well and summarize what we learned, while also reviewing the homeworks.

What we learned can be summarized in the following theorem, sometimes called the Rouché-Capelli theorem.

Theorem. *A system of linear equations has solutions if and only if the rank of its matrix of coefficients equals the rank of its augmented matrix. Moreover, if the system has solutions, then the dimension of the space of solutions is given by the difference between the number of variables and the rank of the matrix of coefficients.*

Moreover, we know that in the case where there is a solution and it is unique, the matrix of coefficients is invertible and the solution can be found by inverting the matrix.

Homework

1. This problem is a review of many of the ideas we learned. Consider the linear function $f : \mathbb{R}^3 \to \mathbb{R}^3$ such that

$$f\left(\begin{bmatrix}1\\1\\1\end{bmatrix}\right) = \begin{bmatrix}1\\6\\11\end{bmatrix},\ f\left(\begin{bmatrix}1\\1\\0\end{bmatrix}\right) = \begin{bmatrix}0\\5\\10\end{bmatrix},\ f\left(\begin{bmatrix}1\\0\\0\end{bmatrix}\right) = \begin{bmatrix}1\\3\\5\end{bmatrix}$$

(a) Compute

$$f\left(\begin{bmatrix}x\\y\\z\end{bmatrix}\right) \text{ and, in particular, } f\left(\begin{bmatrix}2\\3\\4\end{bmatrix}\right).$$

(b) Find the image of this function. If the image is a plane (or a line) find the equation(s) of the plane (or line).

(c) For a given vector $\vec{v}$ in the image set of f, find the inverse image of this vector, $f^{-1}(\vec{v})$. If this is a plane (or a line), find the equation(s) of the plane (or line).