MATH 10
ASSIGNMENT 1: REVIEW OF MATH 9
SEPTEMBER 15, 2019

Welcome to the new semester at SchoolNova!!

Here are the main topics we plan to study this year:

- Linear algebra: systems of linear equations, matrices, and more.
- Beginning abstract algebra: equivalence relations, $\mathbb{Z}_n$, solving equivalences, invertible elements in $\mathbb{Z}_n$, Chinese remainder theorem.
- Beginning analysis: properties of real numbers, limits of sequences.

In this course, we will mainly focus on understanding how these topics actually work. In particular, we will spend most of the time thinking about how to prove that they do work.

We will try to do much of the homework in class so that you do not need to spend too much time on it at home. As usual, all HW assignments and other information will be posted online at `http://www.schoolnova.org`.

I will collect your homework one week after it is assigned and give it back to you with comments a week after. Don't worry if you cannot do all of the problems, but always attempt all of them and write down your progress. Write neatly and in a separate sheet of paper.

I ask that each student bring a notebook (preferably quad ruled), pencils and a folder or binder to keep old assignments — you will need them!

We also plan to participate in math competitions. More details will be given later.

If you have any questions, please contact me by email: `cardoso@schoolnova.org`.

Previous material

I checked the material you covered last year and a lot of it will be useful for us, especially:

Complex numbers.

Mathematical induction.

Sets and functions.

Planar geometry: vectors, dot product.

Today we will review these topics; no new material is given.

Homework

1. Compute

$$\frac{1}{3+4i}$$

2. Let $z = 1 + i\sqrt{3}$.
  (a) Compute z^{2018}.
  (b) Compute $1 + z + z^2 + \cdots + z^{29}$.

3. The polynomial $x^3 - 39x + 70$ has three roots. Two of these roots are $x_1 = 2$, $x_2 = 5$. What is the third root?

4. Prove that for any $n \geq 1$, we have

$$1 + 4 + 7 + \cdots + (3n-2) = \frac{n(3n-1)}{2}$$

5. Let $f\colon A \to B$, $g\colon B \to C$ be injective functions. Prove that then the composition $g \circ f\colon A \to C$ is injective.

(Recall that a function $f\colon A \to B$ is injective (one-to-one) if for any $y \in B$, the equation $f(x) = y$ has at most one solution. Equivalently, f is not injective if there exist $x_1, x_2 \in A$ such that $x_1 \neq x_2$, but $f(x_1) = f(x_2)$.)

6. A subset S of the real line is called *bounded* if one can find an interval $[-M, M]$ which contains all elements of S. Can you write this definition using no words but only quantifiers, variables, arithemtic operations and inequalities?

***7.** (a) Show that, given two arbitrary non-collinear vectors $\vec{a}$ and $\vec{b}$ in the plane, any third vector $\vec{c}$ in the plane can be written as $\vec{c} = x\vec{a} + y\vec{b}$, where the pair of real numbers (x, y) is uniquely defined.

(b) Now suppose that $\vec{a} \cdot \vec{b} = 0$, $\vec{a} \cdot \vec{a} = 1 = \vec{b} \cdot \vec{b}$. Write $\vec{c}_1 \cdot \vec{c}_2$ in terms of the corresponding (x_1, y_1), (x_2, y_2).

(c) If $\vec{c}_2$ is obtained from $\vec{c}_1$ by a counterclockwise rotation by angle ϕ around its base point, how can you express (x_2, y_2) in terms of (x_1, y_1)?