ADVANCED PHYSICS CLUB

MARCH 29, 2020

Problems from last time with solutions

1. Two disks with moments of inertia I_1 and I_2 are rotating around the same vertical axis without friction with angular velocities ω_1 and ω_2 respectively. Disks suddenly come into contact. Because of the friction between the disks after some time there is no relative slipping between the disks. What is the angular velocity of disks then? How much heat was generated during this process?

Solution: Initial angular momenta of disks are $L_1 = I_1\omega_1$ and $L_2 = I_2\omega_2$ respectively. After they come to contact angular momentum is conserved because there are no external torques. Moment of inertia of two disks in contact is $I_{tot} = I_1 + I_2$, so angular momentum conservation tells us $(I_1 + I_2)\omega = I_1\omega_1 + I_2\omega_2$. So, angular velocity after contact is $\omega = \dfrac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2}$. Kinetic energy loss is $Q = \dfrac{1}{2}I_1\omega_1^2 + \dfrac{1}{2}I_2\omega_2^2 - \dfrac{1}{2}(I_1 + I_2)\omega^2 = \dfrac{I_1 I_2(\omega_1 - \omega_2)^2}{2(I_1 + I_2)} \geq 0$. Note that the last expression is manifestly non-negative (energy could only be lost, not gained, due to friction).

Answer: $\omega = \dfrac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2}$, $Q = \dfrac{I_1 I_2(\omega_1 - \omega_2)^2}{2(I_1 + I_2)}$

2. A cylinder of mass m_1 and radius R is at rest on a horizontal plane. A bullet of mass m_2 flying horizontally with velocity v at the height $h < R$ above the cylinder axis hits the cylinder. Assuming the collision is absolutely inelastic and $m_2 \ll m_1$, calculate the axis velocity and angular velocity of the cylinder after the collision.

Solution: Since at the moment of collision there are no unbalanced external forces and torques, we could use both momentum conservation and angular momentum conservation (and energy is not conserved, because the collision is inelastic). Momentum before collision is $p_1 = m_2 v$. $m_2 \ll m_1$, so after collision we could only take into account the momentum of cylinder: $p_2 = m_1 v_0$, where v_0 is the axis velocity. $p_1 = p_2 \Rightarrow v_0 = \dfrac{m_2}{m_1}v$. Angular momentum with respect to the axis of cylinder is $L_1 = m_2 v h$ before the collision and $L_2 = I_1\omega = \dfrac{m_1 R^2}{2}\omega$ after the collision, so $\omega = \dfrac{2m_2 v h}{m_1 R^2}$.

Answer: $v_0 = \dfrac{m_2}{m_1}v, \omega = \dfrac{2m_2 v h}{m_1 R^2}$

3. A man of mass m stands on the edge of a disk, rotating without friction around a vertical axis with angular velocity ω. The disk has radius R and moment of inertia I. How will the angular velocity change if the man moves from the edge to the center of the disk? How will the kinetic energy of the system change? Neglect man's size compared to the disk size.

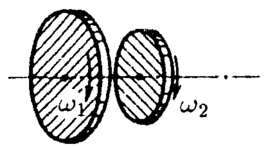
Solution: When man is on the edge, moment of inertia (with respect to the disk axis) of the system "disk+man" is $I_1 = I + mR^2$; after the man walks to the center, it's just $I_2 = I$. From angular momentum conservation $I_2\omega' = I_1\omega$, so $\omega' = \dfrac{I_1}{I_2}\omega = \dfrac{I + mR^2}{I}\omega > \omega$, so angular velocity increases. Kinetic energy after the man walks to the center is $E' = \dfrac{1}{2}I_2\omega'^2 = \dfrac{I + mR^2}{I}E > E$, where $E = \dfrac{1}{2}I_1\omega^2$. So kinetic energy increases (because the man has to perform work to walk to the center).

Answer: $\omega' = \dfrac{I + mR^2}{I}\omega, E' = \dfrac{I + mR^2}{I}E$

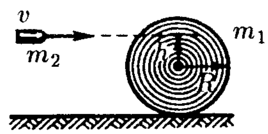
***4.** A uniform rod of length l initially stands at rest vertically on a horizontal frictionless plane. Then it starts to fall. Find the velocity of the top part of the rod just before it hits the surface.

Solution: There are no external horizontal forces, so rod's center of mass is always at the same horizontal coordinate during the fall (vertically it moves down, of course). Just before the top of the rod finally his the ground, the whole rod is just rotating around its' other end, which is no longer moving. Moment of inertia of a uniform rod around its' end is $I_{end} = \frac{ml^2}{3}$, so if we call rod's angular velocity at that moment ω, from energy conservation $\frac{1}{2}I_{end}\omega^2 = \frac{1}{6}ml^2\omega^2 = mg\frac{l}{2}$. Therefore, the velocity of rod's end is $v = \omega l = \sqrt{3gl}$.

Answer: $v = \sqrt{3gl}$



To problem 1



To problem 2

Gyroscopes

One of the most fascinating and counterintuitive applications of the laws of rotation of rigid bodies is to the motion of gyroscope. We encourage you to follow the following links to learn about gyros.

1. General article from Wikipedia:
`https://en.wikipedia.org/wiki/Gyroscope`

2. Couple of slides from hyperphysics:
`http://hyperphysics.phy-astr.gsu.edu/hbase/gyr.html`

3. 6 min video demo on youtube:
`https://www.youtube.com/watch?v=fv_AinDLHJY`

***4.** Lecture by Walter Levin (MIT): Lect 24 - Rolling Motion, Gyroscopes, VERY NON-INTUITIVE:
`https://www.youtube.com/watch?v=XPUuF_dECVI`

Important

This assignment is the last one for this academic year. If you have any physics questions, please write to: apc@schoolnova.org

You can always find this year's materials for APC on SchoolNova's web page:
`https://schoolnova.org/nova/classinfo?class_id=adv_phy_club&sem_id=ay2019`

Stay safe and healthy!
APC Organizers