WEIGHINGS AND ERROR-CORRECTING CODES

APRIL 19, 2020

In these problems, "scale" refers to old-fashioned balance scales, which allow you to compare weighs of items on the left and on the right and determine which one is heavier. It doesn't give the weight of each side - only which of them is heavier.

1. You have 103 coins which look identical. However, two of the coins are fake. It is known that all real coins have the same weight, and both fake coins aslo weigh the same (but different from the real ones).

Can you determine whether the counterfeit coins are lighter or heavier than the real ones in 3 weighings on a balance scales? You are not required to find the counterfeit coins.

2. You have six coins, one of which is fake. It differs in weight from the real ones; however, it is not known whether its is lighter or heavier.

(a) Can you find the fake coin in 3 weighing?

*(b) Can you do the same if you started with 12 coins (also in 3 weighings)?

3. You have 1000 blood samples; it is known that one of them contains a dangerous virus. You have a test that can test for a virus in any sample, but it takes 1 day to complete, and the lab can only run 10 such tests simultaneously (this is limited by the equipment they have). However, the test is so sensitive that he can detect the virus in very small concentrations — for example, if you mix several blood samples together, and one of them contains the virus, then running the test on the mix will show the virus. The test only requires very small amount of material, so each blood sample contains enough for many tests.

Using this, how quickly can you find the infected sample?

4. A detective is working on a murder case. There are 80 people who were present in the area at the time of the murder; one of them is the murderer and one was a witness of the murder. To speed things up, the detective is doing group meetigns, inviting to every meeting some of these 80 people. If the witness is in this group and the murderer is not, the witness tells all to the detective. (If the murderer is there, the witness is too scared to talk.).

Can the detective solve the case in 12 meetings?

Can the detective solve the case in 9 meetings?

***5.** A person selects a number between 1 and 15. Can you find this number by asking seven yes/no questions, if it is known that he can lie once?

Can you do it in 6 questions?