

WEIGHINGS AND GUESSING

MARCH 29, 2020

In these problems, “scale” refers to old-fashioned balance scales, which allow you to compare weights of items on the left and on the right and determine which one is heavier. It doesn’t give the weight of each side - only which of them is heavier.

1. How many questions you need to guess a number between 1-100 using yes/no questions? What about number between 1–1000?
2. You need to design a set of weights that would allow measuring any integer weight from 1 to 15 grams, using the balance scales. What is the smallest set of weights you can come up with? What if you need to measure any integer weight from 1 to 1000 grams?
3. You need to design a set of weights that would allow measuring any integer weight from 1 to 15 grams, using the balance scales (the item to be weighted is placed on one side; the weights can be placed on both sides). What is the smallest set of weights you can come up with? What if you need to measure any integer weight from 1 to 1000 grams?
4. A kindergarten has 21 kids and 21 toy teddy bears. On the first day, each kid was given a teddy bear. On the second, they wanted to switch the teddy bears around so that each kid would get a teddy bear which would be either larger or newer than the one he had on the first day, and at least one kid would get a teddy bear which is both larger and newer than the one he had on the first day. Is it possible?
5. You have 27 coins one of which is fake. It is known that the fake one is lighter than the real ones. How can you find the fake one in 3 weighings using the balance scale? (all real coins weigh the same).
6. You have 63 coins, 7 of which are fake. All real coins weigh the same; all fake coins also weigh the same, but are lighter than the real ones. Can you find 7 coins which are guaranteed to be real in 3 weighings using the balance scale?
7. You have 50 silver coins, of different weights, ordered by weight. You also have 51 gold coins, also ordered by weight.
 - (a) What is the fastest way you can find of arranging all 101 coins in the order of increasing weight, using the balance scale?
 - (b) Can you find the coin which would be 51st by weight among all 101 coins, using only 7 weighings on balance scale?
8. Out of 11 ore samples, 2 are radioactive. Using one measurement, for any set of samples you can find if it has at least one radioactive sample (but can not find how many). Can you find which one is radioactive in 7 measurements?
9. You are given a collection of (different) positive integers. It is known that among any 3 of them you can find two such that one of them divides the other. Prove that then we can divide them in two groups so that for any pair of numbers taken from the same group, one would divide another.
- *10. Let S be a partially ordered set with the following property: in any subset of $d + 1$ element, at least two are comparable. Prove that then S can be partitioned into d subsets so that in each subset, any two elements are comparable. [This is known as Dilworth’s theorem.]