Algebra and Geometry 1. Homework 21.

1. Solve the system of equations;

$$\begin{cases} 4x + y = 2y - 4 \\ 3x + y = x + 1 \end{cases}$$

2. Using the definition of divisibility (if A:B=C, then A=C·B; where A,B, and C are polynomials) prove the following equalities:
   Example:
   $(9x^2 - 4y^2) : (3x + 2y) = 3x - 2y;$
   To prove that we have to multiply $3x - 2y$ by $3x + 2y$ and get $9x^2 - 4y^2$
   $(3x - 2y) \cdot (3x + 2y) = 9x^2 + 6xy - 6xy - 4x^2 = 9x^2 - 4y^2$

   a. $(4a^2 - 20a + 25) : (2a - 5) = 2a - 5$

   b. $\dfrac{3m^3 - 6m^2 - 3m}{m^2 - 2m - 1} = 3m$

   c. $(4a^2 - 11a - 3) : (a - 3) = 4a + 1$

3. For each fraction below find the appropriate set of possible values of variable:

1. $\dfrac{x - 1}{(x - 2)(x - 3)};$ 2. $\dfrac{(x - 2)(x - 3)}{x - 1};$ 3. $\dfrac{x^2}{x^2 + 1}$ 4. $\dfrac{x^2 + 1}{x^2}$

A) $A \neq 1$ *B)* $x \neq 1$ *C)* $x \neq 2, x \neq 3$ *D)* *x can be any number*

4. Write an expression to find the time (t) needed to the boat to sail for S km against the river flow, if the speed of the boat is $v\ km/h$ and speed of the river is $u\ km/h$.
   Find this time for the following values:
   a. $S = 30\ km, v = 10\ km/h, u = 2km/h$
   b. $S = 32\ km, v = 15\ km/h, u = 1km/h$
5. Simplify the fractions and evaluate with given values of variables (hint: simplify first!):

   a. $\dfrac{x^2 - xy + y^2 - (x - y)^2}{x + y};$ $x = 0.3,$ $y = 0.5$

   b. $\dfrac{m - 4}{(m + n)^2 - (m - n)^2};$ $m = \dfrac{2}{3},$ $n = -\dfrac{3}{4}$

   c. $\dfrac{(a + b)^2 - 4ab}{a + b};$ $a = 0.74,$ $b = -0.26$

6. Represent the followimg expressions as an exponent with base 2:
   Example:
$$\left(\frac{16^n}{16^{2n}}\right)^2 = \left(\frac{1}{16^n}\right)^2 = \frac{1}{16^{2n}} = \frac{1}{(2^4)^{2n}} = \frac{1}{2^{8n}} = 2^{-8n}$$

   $a.\ 4^x \cdot 4^y;$ $b.\ 8^x : 8^y;$ $c.\ \left(\left(\frac{1}{4}\right)^x\right)^y;$ $d.\ 4^{-n} \cdot 4^{2n};$ $e.\ ((0.25)^{-2})^n$

7. Find the value:
   $\sqrt{900};$ $\sqrt{6400};$ $\sqrt{810000};$ $\sqrt{250000};$ $\sqrt{16000000};$

   $\sqrt{0.64};$ $\sqrt{0.0064};$ $\sqrt{0.0009};$ $\sqrt{0.000016};$ $\sqrt{0.000004};$
8. Prove that the values of the expressions are natural numbers:
   Example:
   $(\sqrt{3}-1)(\sqrt{3}+1) = \sqrt{3}\sqrt{3} + \sqrt{3} - \sqrt{3} - 1 \cdot 1 = 3 - 1 = 2$
   $a.\ (\sqrt{6}+\sqrt{5})(\sqrt{6}-\sqrt{5});$
   $b.\ (\sqrt{3}+\sqrt{2})^2 + (\sqrt{3}-\sqrt{2})^2;$
   $c.\ (\sqrt{5}-\sqrt{3})^2 + (\sqrt{5}+\sqrt{3})^2;$
   $d.\ (\sqrt{8}+3)^2 - 6\sqrt{8};$