A and G 1. Class work 21.

Algebra.

Completing full square.

Polynomial of the second order can be written generally as:

$$ax^2 + bx + c$$

Where a, b, c are real numbers and x is a variable. A few examples of quadratic polynomials:

$-2x^2 + 5x + 10$

$a^2 + 35 = a^2 + 0 \cdot x + 35$

$7x^2 - \sqrt{33}x - 2$

$4x^2 - 7x + 6$

First, let's move the common factor (fist coefficient) out:

$$ax^2 + bx + c = a\left(\frac{a}{a}x^2 + \frac{b}{a}x + \frac{c}{a}\right) = a\left(x^2 + \frac{b}{a}x + \frac{c}{a}\right)$$

Do you remember the algebraic identity

$$(k+z)^2 = (k+z)(k+z) = k^2 + 2kz + z^2$$

The last expression is

$$\left(x^2 + \frac{b}{a}x + \frac{c}{a}\right)$$

Looks like $k^2 + 2kz + z^2$.

$$\left(x^2 + \frac{b}{a}x + \frac{c}{a}\right) = \left(x^2 + 2 \cdot \frac{1}{2}\frac{b}{a}x + \left(\frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2 + \frac{c}{a}\right)$$

$$= \left(x^2 + 2 \cdot \frac{b}{2a}x + \left(\frac{b}{2a}\right)^2\right) - \left(\left(\frac{b}{2a}\right)^2 - \frac{c}{a}\right)$$

We can further transform the expression:

$$\left(x^2 + 2 \cdot \frac{b}{2a}x + \left(\frac{b}{2a}\right)^2\right) - \left(\left(\frac{b}{2a}\right)^2 - \frac{c}{a}\right) = \left(x + \frac{b}{2a}\right)^2 - \left(\sqrt{\left(\frac{b}{2a}\right)^2 - \frac{c}{a}}\right)^2 =$$

Last part is actually another identity:

$$p^2 - q^2 = (p+q)(p-q):$$

Therefore we can rewrite it

$$\left(x + \frac{b}{2a}\right)^2 - \left(\sqrt{\frac{b^2}{4a^2} - \frac{c}{a}}\right)^2 = \left(x + \frac{b}{2a}\right)^2 - \left(\sqrt{\frac{b^2}{4a^2} - \frac{4ac}{4a^2}}\right)^2 = \left(x + \frac{b}{2a}\right)^2 - \left(\sqrt{\frac{b^2 - 4ac}{4a^2}}\right)^2$$

$$= \left(x + \frac{b}{2a} - \frac{\sqrt{b^2 - 4ac}}{2a}\right)\left(x + \frac{b}{2a} + \frac{\sqrt{b^2 - 4ac}}{2a}\right)$$

Polynomial of the second order can be factorized as

$$ax^2+bx+c=a\left(x+\frac{b}{2a}-\frac{\sqrt{b^2-4ac}}{2a}\right)\left(x+\frac{b}{2a}+\frac{\sqrt{b^2-4ac}}{2a}\right)$$

a, b, c are numbers. Do you think any quadratic polynomial can be factorized?

Factorize:

$4x^2+12x+9=$

$\sqrt{3}x^3-2x-4=$

Exercise:

1. Factorize:

:Example:

$$x^2-15x+50=x^2-10x-5x+50=x(x-10)-5(x-10)=(x-10)(x-5)$$

$$x^2-15x+50=\left(x+\frac{-15}{2}-\frac{\sqrt{225-4\cdot 50}}{2}\right)\left(x+\frac{-15}{2}+\frac{\sqrt{225-4\cdot 50}}{2}\right)$$

$$=\left(x+\frac{-15-\sqrt{225-4\cdot 50}}{2}\right)\left(x+\frac{-15+\sqrt{225-4\cdot 50}}{2}\right)$$

$$=\left(x+\frac{-15-\sqrt{225-4\cdot 50}}{2}\right)\left(x+\frac{-15+\sqrt{225-4\cdot 50}}{2}\right)$$

$$=\left(x+\frac{-15-5}{2}\right)\left(x+\frac{-15+5}{2}\right)=(x-10)(x-5)$$

$a.\ \ 2x^2+9x+4$ $\qquad b.\ \ 3x^2-2x-1$

2. Simplify:

а) $\dfrac{3\sqrt{8}-2\sqrt{12}+\sqrt{20}}{3\sqrt{18}-2\sqrt{27}+\sqrt{45}}$; $\qquad$ б) $\dfrac{\sqrt{28}-2\sqrt{18}-2\sqrt{12}}{6\sqrt{32}+4\sqrt{48}-8\sqrt{7}}$.

а) $(2\sqrt{3}-3\sqrt{2})(2\sqrt{3}+3\sqrt{2})$; $\qquad$ в) $(3\sqrt{5}-2\sqrt{6})^2$;

б) $(\sqrt{32}-3\sqrt{12})(2\sqrt{8}+\sqrt{108})$; $\qquad$ г) $(2-\sqrt{6})^2-(5+\sqrt{2})^2$.

а) $\sqrt{2+\sqrt{3}}\cdot\sqrt{2-\sqrt{3}}$; $\qquad$ в) $\dfrac{\sqrt{\sqrt{30}-\sqrt{5}}}{5}\cdot\dfrac{\sqrt{\sqrt{30}+\sqrt{5}}}{5}$;

б) $\sqrt{4-\sqrt{7}}\cdot\sqrt{4+\sqrt{7}}$; $\qquad$ г) $\dfrac{\sqrt{\sqrt{3}+\sqrt{15}}}{2}\cdot\dfrac{\sqrt{\sqrt{15}-\sqrt{3}}}{3}$.