A and G 1. Class work 19.

Algebra.

Evaluate:

$\sqrt{4}$	$\sqrt{100}$	$\sqrt{0.49}$
$\sqrt{36}$	$\sqrt{16}$	$\sqrt{0.09}$
$\sqrt{1}$	$\sqrt{0.01}$	$\sqrt{0.0064}$
$\sqrt{\frac{1}{81}}$	$\sqrt{\frac{4}{25}}$	$\sqrt{\frac{400}{49}}$
$\sqrt{\frac{1}{100}}$	$\sqrt{\frac{9}{64}}$	$\sqrt{\frac{16}{9}}$

Base on the definition of arithmetic square root we can right

$$\left(\sqrt{a}\right)^2 = a$$

To keep our system of exponent properties consistent let's try to substitute $\sqrt{a} = a^k$. Therefore,

$$\left(\sqrt{a}\right)^2 = (a^k)^2 = a^1$$

But we know that

$$(a^k)^2 = a^{2k} = a^1 \quad \Rightarrow \quad 2k = 1, \;\; k = \frac{1}{2}$$

And we can agree to consider arithmetic square root as fractional exponent

$$\sqrt{a} = a^{\frac{1}{2}}$$

$$a = \left(\sqrt{a}\right)^2 = \left(a^{\frac{1}{2}}\right)^2 = a^{\frac{1}{2}\cdot 2} = (a^2)^{\frac{1}{2}} = \sqrt{a^2} = a$$

$$\left(\sqrt{a}\right)^2 = a, \qquad \left(\sqrt{b}\right)^2 = b,$$

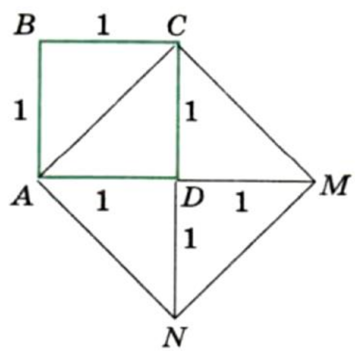
$$\left(\sqrt{a}\right)^2\left(\sqrt{b}\right)^2 = \left(\sqrt{a}\sqrt{b}\right)^2 = ab = \left(\sqrt{ab}\right)^2 \Rightarrow \sqrt{a}\sqrt{b} = \sqrt{ab}$$

Properties of arithmetic square root:

For any number a the following properties are true;

1. $\sqrt{a^2} = |a|$
2. if $a > b > 0, \sqrt{a} > \sqrt{b}$
3. for $a \geq 0,\ b \geq 0, \sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$
4. for $a \geq 0,\ b \geq 0, \sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

To solve equation $x^2 = 23$ we have to find two sq. root of 23. $x = \pm\sqrt{23}$. 23 is not a perfect square as 4, 9, 16, 25, 36 …



The length of the segment [AC] is $\sqrt{2}$ (from Pythagorean theorem). The area of the square ACMN is twice the area of the square ABCD. Let assume that the $\sqrt{2}$ is a rational number, so it can be represented as a ratio $\frac{p}{q}$, where $\frac{p}{q}$ is nonreducible fraction.

$$\left(\frac{p}{q}\right)^2 = 2 = \frac{p^2}{q^2}$$

Or $p^2 = 2q^2$, therefore p^2 is an even number, and p itself is an even number, and can be represented as $p = 2p_1$., consequently

$p^2 = (2p_1)^2 = 4{p_1}^2 = 2q^2$

$2{p_1}^2 = q^2 \Rightarrow q$ also is an even number and can be written as $q = 2q_1$.

$\frac{p}{q} = \frac{2p_1}{2q_1}$, therefore fraction $\frac{p}{q}$ can be reduced, which is contradict the assumption. We proved that the $\sqrt{2}$ isn't a rational number by contradiction.

Exercises:

1. Simplify;

$\sqrt{2^{10000}}$; $\sqrt{3^{-50}}$; $\sqrt{7^{500}}$; $\sqrt{5^{-100}}$

2. Evaluate:

   $\sqrt{\sqrt{81}}$; $\sqrt{\sqrt{625}}$; $\sqrt{11 + \sqrt{25}}$; $\sqrt{\sqrt{49} - \sqrt{36}}$

3. Euler formula for prime numbers:
   $n^2 - n + 41$ is a prime number for any $n \in N$. Prove or disapprove it.

4. Compute:

$$\frac{10^2 + 11^2 + 12^2 + 13^2 + 14^2}{365} =$$