

A and G 1. Class work 18.  
**Algebra.**



$$\frac{(999^{-1} - 1000^{-1})(999^{-1} + 1000^{-1})}{(1000^{-1} - 999^{-1})^2} =$$

Rational number is a number which can be represented as a ratio of two integers:

$$a = \frac{p}{q}; \quad p \in \mathbb{Z}, \text{ and } q \in \mathbb{N}, \quad (\mathbb{Z} = \{\pm \dots, \pm 1, 0\}, \mathbb{N} = \{1, 2, \dots\})$$

Rational numbers can be represented as infinite periodical decimals (in the case of denominators containing only powers of 2 and 5 the periodical bloc of such decimal is 0).

Numbers, which can't be express as a ratio (fraction)  $\frac{p}{q}$  for any integers  $p$  and  $q$  are irrational numbers. Their decimal expansion is not finite, and not periodical.

Examples:

0.01001000100001000001...

0.123456789101112131415161718192021...

What side the square with the area of  $a$  m<sup>2</sup> does have? To solve this problem, we have to find the number, which gives us  $a$  as its square. In other words, we have to solve the equation

$$x^2 = a$$

This equation can be solved (has a real number solution) only if  $a$  is nonnegative ( $(a \geq 0)$ ) number. It can be seen very easily;

$$\text{If } x = 0, \quad x \cdot x = x^2 = a = 0,$$

$$\text{If } x > 0, \quad x \cdot x = x^2 = a > 0,$$

$$\text{If } x < 0, \quad x \cdot x = x^2 = a > 0,$$

We can see that the square of any real number is a nonnegative number, or there is no such real number that has negative square.

**Square root** of a (real nonnegative) number  $a$  is a number, square of which is equal to  $a$ .

There are only 2 square roots from any positive number, they are equal by absolute value, but have opposite signs. The square root from 0 is 0, there is no any real square root from negative real number.

Examples:

1. Find square roots of 16: 4 and  $(-4)$ ,  $4^2 = (-4)^2 = 16$
2. Numbers  $\frac{1}{7}$  and  $(-\frac{1}{7})$  are square roots of  $\frac{1}{49}$ , because  $\frac{1}{7} \cdot \frac{1}{7} = (-\frac{1}{7}) \cdot (-\frac{1}{7}) = \frac{1}{49}$
3. Numbers  $\frac{5}{3}$  and  $(-\frac{5}{3})$  are square roots of  $\frac{25}{9}$ , because  $(\frac{5}{3})^2 = \frac{5}{3} \cdot \frac{5}{3} = (-\frac{5}{3})^2 = (-\frac{5}{3}) \cdot (-\frac{5}{3}) = \frac{25}{9}$

**Arithmetic square root** of a (real nonnegative) number  $a$  is a nonnegative number, square of which is equal to  $a$ .

There is a special sign for the arithmetic square root of a number  $a$ :  $\sqrt{a}$ .

Examples;

1.  $\sqrt{25} = 5$ , it means that arithmetic square root of 25 is 5, as a nonnegative number, square of which is 25. Square roots of 25 are 5 and  $(-5)$ , or  $\pm\sqrt{25} = \pm 5$
2. Square roots of 121 are 11 and  $(-11)$ , or  $\pm\sqrt{121} = \pm 11$
3. Square roots of 2 are  $\pm\sqrt{2}$ .
4. A few more:

$$\begin{array}{lllll} \sqrt{0} = 0; & \sqrt{1} = 1; & \sqrt{4} = 2; & \sqrt{9} = 3; & \sqrt{16} = 4; \\ \sqrt{25} = 5; & \sqrt{\frac{1}{64}} = \frac{1}{8}; & \sqrt{\frac{36}{25}} = \frac{6}{5} & & \end{array}$$

Base on the definition of arithmetic square root we can right

$$(\sqrt{a})^2 = a$$

To keep our system of exponent properties consistent let's try to substitute  $\sqrt{a} = a^k$ . Therefore,

$$(\sqrt{a})^2 = (a^k)^2 = a^1$$

But we know that

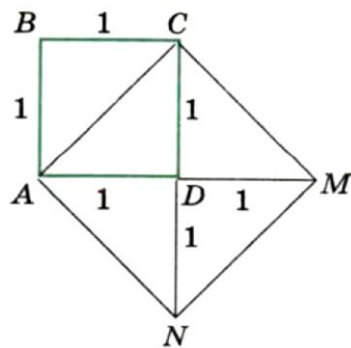
$$(a^k)^2 = a^{2k} = a^1 \Rightarrow 2k = 1, k = \frac{1}{2}$$

And we can agree to consider arithmetic square root as fractional exponent

$$\sqrt{a} = a^{\frac{1}{2}}$$

$$(\sqrt{a})^2 = a, \quad (\sqrt{b})^2 = b, \quad (\sqrt{a})^2 (\sqrt{b})^2 = (\sqrt{a}\sqrt{b})^2 = ab = (\sqrt{ab})^2 \Rightarrow \sqrt{a}\sqrt{b} = \sqrt{ab}$$

To solve equation  $x^2 = 23$  we have to find two sq. root of 23.  $x = \pm\sqrt{23}$ . 23 is not a perfect square as 4, 9, 16, 25, 36 ...



The length of the segment [AC] is  $\sqrt{2}$  (from Pythagorean theorem). The area of the square ACMN is twice the area of the square ABCD. Let assume that the  $\sqrt{2}$  is a rational number, so it can be represented as a ratio  $\frac{p}{q}$ , where  $\frac{p}{q}$  is nonreducible fraction.

$$\left(\frac{p}{q}\right)^2 = 2 = \frac{p^2}{q^2}$$

Or  $p^2 = 2q^2$ , therefore  $p^2$  is an even number, and  $p$  itself is an even number, and can be represented as  $p = 2p_1$ ., consequently

$$p^2 = (2p_1)^2 = 4p_1^2 = 2q^2$$

$2p_1^2 = q^2 \Rightarrow q$  also is an even number and can be written as  $q = 2q_1$ .

$\frac{p}{q} = \frac{2p_1}{2q_1}$ , therefore fraction  $\frac{p}{q}$  can be reduced, which is contradict the assumption. We proved that the  $\sqrt{2}$  isn't a rational number by contradiction.

Prove that the value of the following expressions is a rational number.

1.  $(\sqrt{2} - 1)(\sqrt{2} + 1)$
2.  $(\sqrt{5} - \sqrt{3})(\sqrt{5} + \sqrt{3})$
3.  $(\sqrt{2} + 1)^2 + (\sqrt{2} - 1)^2$
4.  $(\sqrt{7} - 1)^2 + (\sqrt{7} + 1)^2$
5.  $(\sqrt{7} - 2)^2 + 4\sqrt{7}$

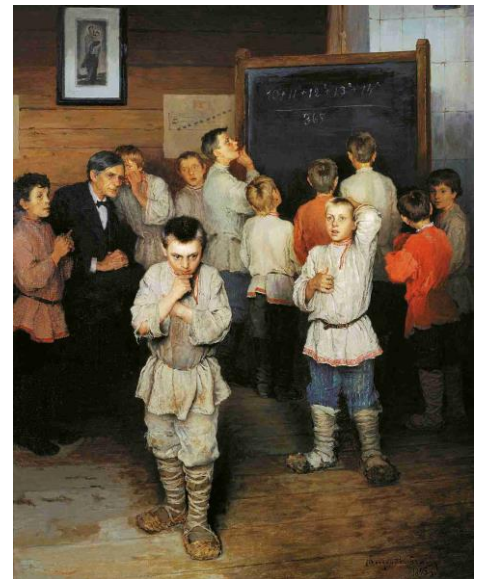
- a)  $\sqrt{12}$ ;    б)  $\sqrt{18}$ ;    в)  $\sqrt{20}$ ;    г)  $\sqrt{24}$ ;    д)  $\sqrt{27}$ ;  
 е)  $\sqrt{28}$ ;    ж)  $\sqrt{32}$ ;    з)  $\sqrt{45}$ ;    и)  $\sqrt{50}$ ;    к)  $\sqrt{72}$ .

### Exercises:

1. Euler formula for prime numbers:  
 $n^2 - n + 41$  is a prime number for any  $n \in \mathbb{N}$ . Prove or disapprove it.
2. Compute:

$$\frac{10^2 + 11^2 + 12^2 + 13^2 + 14^2}{365} =$$

3. Simplify:



$$a. \frac{5x - 16}{x^2 - 9};$$

$$c. \frac{a^2 - 5a + 6}{3a^2 - 6a};$$

$$b. \frac{3x^2 + 14x - 5}{3x^2 + 2x - 1}$$

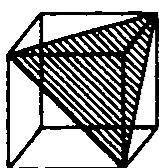
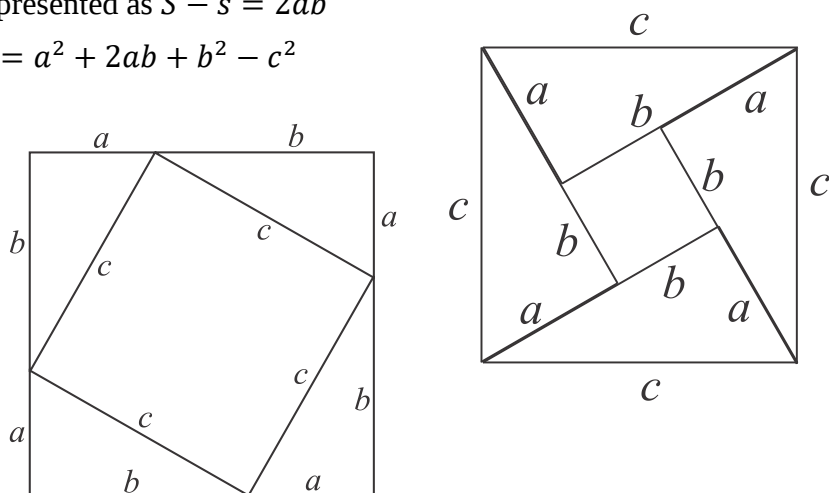
$$d. \frac{5p - 10}{p^2 - 4}$$

### Geometry.

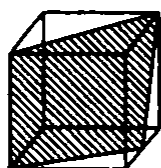
### Pythagorean theorem.

4 identical right triangles are arranged as shown on the picture. The area of the big square is  $S = (a + b) \cdot (a + b) = (a + b)^2$ , the area of the small square is  $s = c^2$ . The area of 4 triangles is  $4 \cdot \frac{1}{2}ab = 2ab$ . But also can be represented as  $S - s = 2ab$   
 $2ab = (a + b) \cdot (a + b) - c^2 = a^2 + 2ab + b^2 - c^2$   
 $\Rightarrow a^2 + b^2 = c^2$

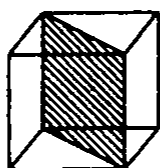
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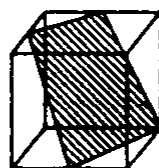
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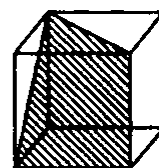
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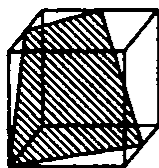
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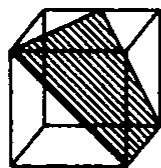
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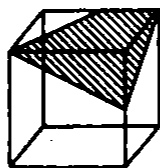
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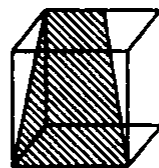
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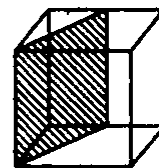
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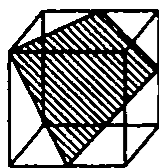
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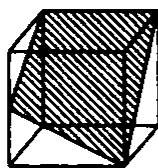
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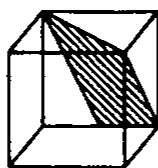
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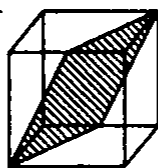
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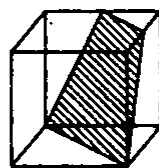
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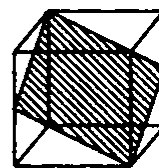
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