A and G 1. Class work 12.

Algebra.

Last week we learn about logical statements and logical operation; NOT, AND, OR (XOR).

AND: For statements A and B

A	**B**	**$A \wedge B$ (AND, ·)**
T	T	T
T	F	F
F	T	F
F	F	F

OR (inclusion OR): For statements A and B

A	**B**	**$A \vee B$ (OR, +)**
T	T	T
T	F	T
F	T	T
F	F	F

XOR (inclusion OR): For statements A and B

A	**B**	**A XOR B**
T	T	F
T	F	T
F	T	T
F	F	F

NOT: for the statement (predicate) A

A	**$\neg A$**
T	F
F	T

How we can construct the negation of the complex statement? Remember, that the negated statement should be true if direct statement is false and vice versa.

$\neg$(A AND B) is equivalent to ($\neg$A OR $\neg$B) and $\neg$(A OR B) is equivalent to $\neg$A AND $\neg$B.

A	**B**	**$A \wedge B$**	**$\neg (A \wedge B)$**	**$\neg A$**	**$\neg B$**	**$\neg A \vee \neg B$**
T	T	T	F	F	F	F
T	F	F	T	F	T	T
F	T	F	T	T	F	T
F	F	F	T	T	T	T

Another logical operation: $\Rightarrow$ “implies", or “if A then B", “from A follows B”.

A	**B**	**$A \Rightarrow B$**
T	T	T
T	F	F
F	T	T
F	F	T

If the statement A is true and B is true, then we can say that “if A then B" is a true statement. Obviously, if B is not true, then A does not imply B, and the whole statement is false. Unintuitively, if A false, whole statement is true. From the false statement anything can follow. Two statements A ⇒ B and B ⇒ A are not equivalent. For example:
Mather is telling her son: “If you do your room you will play a videogame”. Of cause, it doesn’t mean that if the son plays the videogame, he immediately will do his room. But there is a contrapositive rule: A ⇒ B is equivalent to (¬B) ⇒ (¬A). (the real life example above then should be read as “if son don’t play videogames he will not do his room”, which is not always the case).
Math example is more relevant:
If the natural number is divisible by 6, it’s divisible by 3.
Statement A: “natural number is divisible by 6”, statement B: “natural number is divisible by 3”.
If A then B: true if A is true and B is true. Can be proven by:
If the natural number disable by 6 it can be represented as

$$6n, \quad n \epsilon N$$

$$6n = 3 \cdot 2 \cdot n = 3 \cdot (2n)$$

But the statement B ⇒ A is not true, if the number is divisible by 3, it’s not necessarily divisible by 6. But if the number is not divisible by 3 (¬B), then it is definitely not divisible by 6 (¬A). ((¬B) ⇒ (¬A)).
From A follows B, from NOT B follows NOT A.
This construction is very useful in deducing new results from known ones. Here are some of the rules:
• Given A ⇒ B and B ⇒ C, we can conclude A ⇒ C
• Given A ⇒ B and NOT B, we can conclude NOT A

Knowing all this logic rules we can construct more complex statements:
To get a + for my homework I need to do at least one of the assignments, solve one math and one physics problems or write an essay.
Statement A: solve math problem
Statement B: solve physics problem
Statement C: write an essay
(A AND B) OR C

A	B	A AND B	C	(A AND B) OR C
T	T	T	T	T
T	F	F	T	T
F	T	F	T	T
F	F	F	T	T
T	T	T	F	T
T	F	F	F	F
F	T	F	F	F
F	F	F	F	F

Many of the questions of this assignment refer to the famous (among logic puzzle fans) island of Knights and Knaves. On this island, there are two kinds of people: Knights, who always tell the truth, and Knaves, who always lie. Unfortunately, there is no easy way of knowing whether a person you meet is a knight or a knave... Copyright notice: most of these problems come from books of Raymond Smullyan.

1. On the island of knights and knaves, you meet two inhabitants: Zoey and Mel. Zoey tells you that Mel is a knave. Mel says, “Neither Zoey nor I are knaves." So who is a knight and who is a knave?

2. On the island of knights and knaves, you meet two inhabitants: Sue and Zippy. Sue says that Zippy is a knave. Zippy says, “I and Sue are knights." So who is a knight and who is a knave?

3. Many trucks carry the message: “If you do not see my mirrors, then I do not see you". Can you rewrite it in an equivalent form without using the word “not"?

4. Check whether $A \Rightarrow B$ and $B \Rightarrow A$ are equivalent, by writing the truth table for each of them.

5. Check that $A \Rightarrow B$ is equivalent to (NOT A) OR B (thus, “if you do not clean up your room, you will be punished" and “clean up your room, or you will be punished" are the same).

6. A mom tells the son “If you do not do the dishes, you will not go to the movie". Is it the same as “If you do the dishes, you go to the movie?"

7. Write the truth table for each of the following formulas. Are they equivalent (i.e., do they always give the same value)?
(a) (A AND B) OR (A AND C)
(b) A AND (B OR C).

8. If today is Thursday, then Jane's class has library day. If Jane's class has library day, then Jane will bring home new library books. Jane brought no new library books. Therefore,. . .

9. If it is Tuesday and Bill is in a good mood, he goes to his favorite pub, and when he goes to his favorite pub, he comes home very late. Today Bill came home early. Therefore, . . .

Geometry:
Three congruent rectangles on the picture form the figure. What is the angle? Is it possible for any rectangle?