Algebra.

1. The products of inside and outside terms are equal.
$$\frac{a}{b} = \frac{c}{d} \quad a \cdot d = b \cdot c$$

It can be easily shown:
$$\frac{a}{b} = \frac{c}{d} \quad \frac{adb}{b} = \frac{cdb}{d} \quad ad = cb$$

inside terms

$$\frac{a}{b} = \frac{c}{d} \quad \text{or} \quad a : b = c : d$$

outside terms

2. Also, two inverse ratios are equal:
$$\frac{a}{b} = \frac{c}{d} \quad \frac{b}{a} = \frac{d}{c}$$

Indeed:
$$\frac{a}{b} = \frac{c}{d} \quad a \cdot d = b \cdot c \quad \frac{ad}{ac} = \frac{bc}{ac} \quad \frac{d}{c} = \frac{b}{a}$$

3. Two outside terms can be switched:
$$\frac{a}{b} = \frac{c}{d} \quad \frac{d}{b} = \frac{c}{a}$$
$$\frac{a}{b} = \frac{c}{d} \quad a \cdot d = b \cdot c \quad \frac{ad}{ab} = \frac{bc}{ab} \quad \frac{d}{c} = \frac{b}{a}$$

4. Two inside terms can be switched as well.
$$\frac{a}{b} = \frac{c}{d} \quad \frac{c}{d} \quad \frac{a}{c} = \frac{b}{d}$$

5. Several other proportions can be constructed.
$$\frac{a}{b} = \frac{c}{d} \quad \frac{a \pm b}{b} = \frac{c \pm d}{d}$$

(the sign $\pm$ is used to show that both, addition and subtraction, can be used)
Let's prove one of the statements:

$$\frac{a}{b} = \frac{c}{d} \quad \frac{a}{b} + 1 = \frac{c}{d} + 1 \quad \frac{a}{b} + \frac{b}{b} = \frac{c}{d} + \frac{d}{d} \quad \frac{a + b}{b} = \frac{c + d}{d}$$

6. Another proportion:
$$\frac{a}{b} = \frac{c}{d} \quad \frac{a + c}{b + d} = \frac{c}{d} = \frac{a}{b}$$

It can be proved as follow:

$$\frac{a+c}{b+d} = \frac{a\left(1+\frac{c}{a}\right)}{b\left(1+\frac{d}{b}\right)}$$

We know from (4) that

$$\frac{c}{a} = \frac{d}{b}$$

$$\frac{a+c}{b+d} = \frac{a\left(1+\frac{c}{a}\right)}{b\left(1+\frac{d}{b}\right)} = \frac{a}{b}$$

Proportionality.

A car is driving at a constant speed 50 km/hour along a straight highway. How far it will go in 1 hour? 2 hours, 5 hours? 15 hours?
Fill the table:

Time (t)	1 hour	2 hours	5 hours	15 hours
Distance (d)				

What did you notice?

$$\frac{d_1}{t_1} = \frac{d_2}{t_2} = \frac{d_3}{t_3} = \frac{d_4}{t_4} = 50 \; km/h$$

In other words, $d = 50\left(km/h\right) * t(hour)$. Here the number 50 is called a constant of proportionality. Longer travel $\Rightarrow$ further from the initial point and the ratio between these to variables will be always the same, *distance:time* is speed. If the time is three times longer, the distance will be three times greater as well. (When the speed remains constant, of course). Two variables, *distance* and *time,* are dependent on each other; they are in the relation of proportionality. Let's draw the graph in your notebooks, *abscissa* will be the time, and *ordinate* will be the distance. We can write the relationship between the distance, time, and speed as

$$S = v \cdot t$$

A notebook costs 3 dollars. How much do I need to pay if I am buying 3 notebooks, 5, 12? What variables are used, what is the relationship between them? Can a constant of proportionality be found?

Two other problems.

1. A squirrel is storing acorns for winter. Every 20 minutes it brings 2 acorns. How many acorns will there be in 40 minutes? 80 minutes?

Time (t)	20 minutes	40 minutes	80 minutes	2 hours
Number of acorns				

2. Bacteria are dividing every 20 minutes. I want to make yogurt and I put 1 bacterium in a cup of milk. How many bacteria will be there in the milk in 20 minutes? in 40 minutes? in 80 minutes?

Time (t)	20 minutes	40 minutes	80 minutes	2 hours
Number of bacteria				

3.

In which problem the variables are proportional?

Inverse proportionality.

Consider two cities, A and B, with the distance between them equal to 10 km. Compute the speed needed to get from A to B in a given time (fill out the table).

Time (t)	1 hour	2 hours	0.5 hours	0.1 hours
Speed (v)				

In this case, we see an example of the inverse proportionality. If the speed is twice lower, one needs a two times longer time to get to the city B. If time is increased 2 times, the speed is decreased 2 times.

Exercises.

1. Solve the following equations (hint: use proportionality)

$$\frac{x - \frac{2}{7}}{\frac{2}{7}} = \frac{48.3}{0.7}; \frac{1.8}{6.8} = \frac{0.042}{1\frac{1}{6}x + 0.042}$$

2. The relationship between the two variables is given in the table below. Is this relationship proportional? If so, what is the constant of proportionality?

a.

x	9	15	33	45	66
y	3	5	11	15	22

b.

x	3	2	5	4	6
y	9	4	25	16	36

c.

x	3	2	1	$\frac{1}{3}$	30
y	1	$\frac{3}{2}$	3	9	0.1

3. Are the following variables proportional?
   a. Speed and time of movement on a distance of 50 km.
   b. Speed and corresponding distance after 2 hours of driving.
   c. Price of the 1 notebook and the number of notebooks that can be bought with 24 dollars.
   d. Length and width of the rectangle with the area of 60 cm^2.

4. A car travels 60 km during a certain time. How time of travel change, if the speed is increased 3 times?

5. Which of the following formulas describe the direct proportionality, inverse proportionality, neither of the two?

$$P = 5.2b; \quad K = \frac{n}{2}; \quad a = \frac{8}{b}; M \quad = m: 5; \quad G = \frac{1}{4k};$$
$$a = 8q + 1; \quad c = 4: d, \quad 300 = v \cdot t; \quad ab = 18; \quad S = a^2$$

6. Peter's time of driving to work is usually 1 hour and 20 minutes. Yesterday the weather was bad and Peter reduced his speed by 10 km/h and reached his work in 1.5 hours. What is the distance between Peter's work and his house?

7. In a driving school, a car with an instructor and three students went to the ride. The instructor drove $\frac{2}{15}$ of the whole distance and 5 km, two students drove $\frac{1}{4}$ of the distance each, and the third students drove the remaining 105 km. What was the length of the whole itinerary?

Geometry.
Pythagorean theorem.
$$a^2 + b^2 = c^2$$

4 identical right triangles are arranged as shown in the figure.
The area of the big square is
$$S = (a + b) \cdot (a + b) = (a + b)^2,$$
the are of the small square is
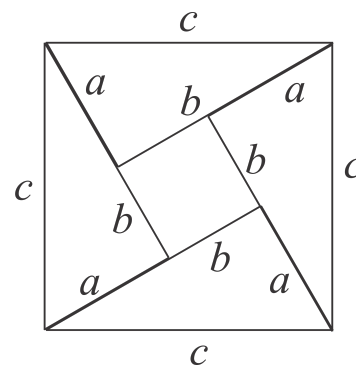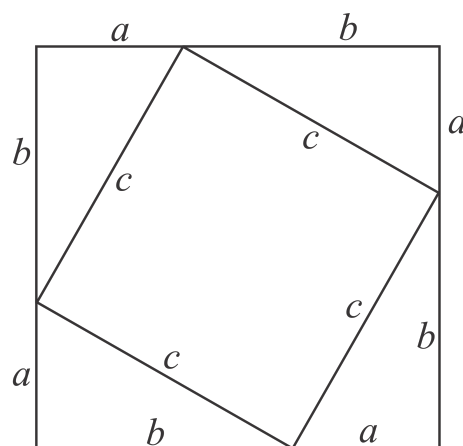$$s = c^2,$$
The total area of 4 triangles is
$$4 \cdot \frac{1}{2} ab = 2ab.$$
This area can also be represented as $S - s$. We have
$$2ab = S - s,$$
$$2ab = (a + b) \cdot (a + b) - c^2 = a^2 + 2ab + b^2 - c^2,$$
$$a^2 + b^2 = c^2$$

Exercises:

8. Legs of a right triangle are 3 cm and 4 cm. What is the length of its hypotenuse?

9. The perimeter of a given triangle is 36 cm. The lengths of the sides are in ratio 2:3:4. What are the lengths of all sides? Draw the triangle with these sides.