A and G 1. Class work 8

Algebra.

In mathematics, a **ratio** is a relationship between two numbers indicating how many times the first number contains the second. For example, if a bowl of fruit contains eight oranges and six lemons, then the ratio of oranges to lemons is eight to six (that is, $8:6$, which is equivalent to the ratio $4:3$). Similarly, the ratio of lemons to oranges is $6:8$ (or $3:4$) and the ratio of oranges to the total amount of fruit is $8:14$ (or $4:7$).

The numbers in a ratio may be quantities of any kind, such as counts of persons or objects, or such as measurements of lengths, weights, time, etc. In most contexts both numbers are restricted to be positive, and the second to be not zero.

Ratios are sometimes used with three or even more terms, for example, a good concrete mix (in volume) is

$$cement : send : gravel \;=\; 1:2:4$$

It means that for 1 volume part of cement go 2 volume parts of send and 4 volume parts of gravel.

In a nut mix the ratio of nuts is

$$walnats : almonds : haselnuts = 3:5:7$$

The bag of this mix weights 1500 g. What is the weight of almonds?

If a mixture contains substances A, B, C and D in the ratio $5:9:4:2$ then there are 5 parts of A for every 9 parts of B, 4 parts of C and 2 parts of D. As $5+9+4+2=20$, the total mixture contains $5/20$ of A (5 parts out of 20), $9/20$ of B, $4/20$ of C, and $2/20$ of D. If we divide all numbers by the total and multiply by 100, we have converted to percentages: 25% A, 45% B, 20% C, and 10% D (equivalent to writing the ratio as $25:45:20:10$).

To cook a raspberry jam according to recipe I need to combine three cups of berries and 2 cups of sugar, or for each 3 cups of raspberries go 2 cups of sugar; ratio of raspberries and sugar (in volume) is $3:2$. If I bought 27 cups of raspberries, how many cups of sugar do I need to put to my jam? Or I have only 5 cups of sugar and I don't want to go to the store, how many cups of raspberries I need to pick in my garden to make the same tasty jam? The ratio should stay the same:

$$\frac{3}{2}=\frac{27}{x}=\frac{y}{5}$$

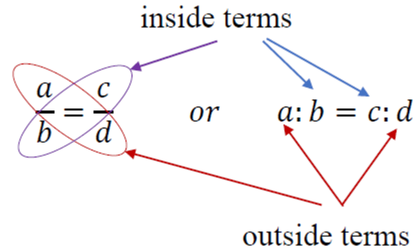
Two ratios which are equal form a proportion. In the case of raspberry jam, me have two proportions:

$$\frac{3}{2}=\frac{27}{x}, \text{and } \frac{3}{2}=\frac{y}{5}$$

x is number of cups of sugar for 27 cups of raspberries, and y is the number of cups of raspberries needed for 5 cups of sugar.

Proportions have several interesting features.

1. The products of inside and outside terms are equal.
$$\frac{a}{b}=\frac{c}{d} \quad \Leftrightarrow \quad a\cdot d=b\cdot c$$
It can be easily shown:
$$\frac{a}{b}=\frac{c}{d}\Leftrightarrow \frac{adb}{b}=\frac{cdb}{d}\Leftrightarrow ad=cb$$



2. Also, two inverse ratios are equal:
$$\frac{a}{b}=\frac{c}{d} \quad \Leftrightarrow \quad \frac{b}{a}=\frac{d}{c}$$

Indeed:
$$\frac{a}{b}=\frac{c}{d} \quad \Leftrightarrow \quad a\cdot d=b\cdot c \quad \Leftrightarrow \quad \frac{ad}{ac}=\frac{bc}{ac} \quad \Leftrightarrow \quad \frac{d}{c}=\frac{b}{a}$$

3. Two outside terms can be switched:
$$\frac{a}{b}=\frac{c}{d} \quad \Leftrightarrow \quad \frac{d}{b}=\frac{c}{a}$$
$$\frac{a}{b}=\frac{c}{d} \quad \Leftrightarrow \quad a\cdot d=b\cdot c \quad \Leftrightarrow \quad \frac{ad}{ab}=\frac{bc}{ab} \quad \Leftrightarrow \quad \frac{d}{c}=\frac{b}{a}$$
4. Two inside terms can be switched as well.
$$\frac{a}{b}=\frac{c}{d} \quad \Leftrightarrow \quad \frac{a}{c}=\frac{b}{d}$$
5. Also, several other new proportion can be created.
$$\frac{a}{b}=\frac{c}{d} \quad \Leftrightarrow \quad \frac{a\pm b}{b}=\frac{c\pm d}{d}$$
(the sign $\pm$ is used to show that both, addition and subtraction, can be used)
Let's prove one of the statements:

$$\frac{a}{b}=\frac{c}{d} \quad \Leftrightarrow \quad \frac{a}{b}+1=\frac{c}{d}+1 \quad \Leftrightarrow \quad \frac{a}{b}+\frac{b}{b}=\frac{c}{d}+\frac{d}{d} \quad \Leftrightarrow \quad \frac{a+b}{b}=\frac{c+d}{d}$$
6. Another proportion:
$$\frac{a}{b}=\frac{c}{d} \quad \Leftrightarrow \quad \frac{a+c}{b+d}=\frac{c}{d}=\frac{a}{b}$$
It can be proved as follow:
$$\frac{a+c}{b+d}=\frac{a\left(1+\frac{c}{a}\right)}{b\left(1+\frac{d}{b}\right)}$$
We know form (4) that
$$\frac{c}{a}=\frac{d}{b}$$

$$\frac{a+c}{b+d}=\frac{a\left(1+\frac{c}{a}\right)}{b\left(1+\frac{d}{b}\right)}=\frac{a}{b}$$

Going back to the jam problem above. We got the simple equation

$$\frac{3}{2}=\frac{27}{x}$$

It can be solved easily using the property of proportion

$$3x=27\cdot 2$$

$$x=\frac{27\cdot 2}{3}=\frac{3\cdot 9\cdot 2}{3}=18$$

Now let’s take a look on the motion of the object. If the object is moving with the constant speed, The ratio of the distance it moved in a certain time and the time will be always the same -speed. Indeed, if the car went 100 km in 2 hours, it means that each hour it passed

$$\frac{S(km)}{t(h)}=v({}^{km}/_{h})$$

Or

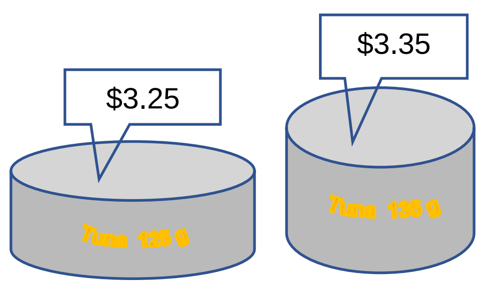
$$S(km)=v\left({}^{km}/_{h}\right)t(h)$$

Exercises:

1. Solve the following equations (hint: use the property of proportions):

$$a.\ \ \frac{x}{7.2}=\frac{1\frac{1}{9}}{0.25};\qquad b.\ \ \frac{2\frac{1}{3}}{0.6x}=\frac{2.5}{1\frac{2}{7}};\qquad c.\ \ \frac{\frac{7}{12}}{0.14}=\frac{50x}{4.8};\qquad d.\ \ \frac{1\frac{3}{17}}{13.75}=\frac{2\frac{2}{11}}{3x}$$

2. A company packs tuna into 2 different type of cans, 125 g and 135 g. 125 g can costs \$3.25 and 135 g can costs \$3.35. In which can tuna is less expensive?



3. John and Robert played basketball. John made 20 throws and hit 15 times. Robert made 27 throws and hit 18 times. Who did better?
4. The ratio of boys to girls in 6th grade is $\frac{9}{11}$. The ratio of girls to boys in 7th grade is $\frac{31}{29}$. There are 100 and 120 students in 6th and 7th grades correspondingly, what is a ratio of boys to girls at the dance for 6 and 7 grade students, if all students came to the dunce.
5. Evaluate:

$$\frac{(2.3+5.8)\cdot 3\frac{5}{7}}{(4.9-2.3):\frac{7}{9}}\quad (answer\ is\ 9);\qquad\qquad \frac{\frac{1}{8}:\frac{5}{16}+2.25\cdot 0.8}{\left(2\frac{1}{48}-1\frac{55}{72}\right):3\frac{1}{12}}+3\frac{3}{5}\quad (answer\ is\ 30)$$

6. By how many percent A is greater than B

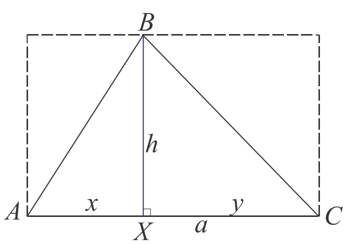
$$A = \frac{\left(7\frac{3}{8} - 2.125\right) \cdot 2\frac{2}{7} - 39.48 : 5.6}{(3.4 \cdot 0.9 - 2.7) : 0.06 \cdot 2\frac{2}{3} - 30.9 \cdot 0.5}$$

$$B = \frac{\left(6.1 \cdot 3.05 - 2.05 \cdot \left(4\frac{3}{5} + 4.46\right)\right) \cdot 22.5}{\left(1\frac{1}{4} + 0.5 + 2\frac{1}{3}\right) : 2\frac{1}{24} \cdot 0.01}$$

Geometry.

Area of a triangle.

The area of a triangle is equal to half of the product of its altitude and the base, corresponding to this altitude.

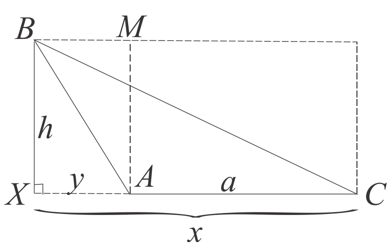


For the acute triangle it is easy to see.

$$S_{rec} = h \times a = x \times h + y \times h$$

$$S_{\Delta ABX} = \frac{1}{2} h \times x, \qquad S_{\Delta XBC} = \frac{1}{2} h \times y, \qquad S_{\Delta ABC} = S_{\Delta ABX} + S_{\Delta XBC}$$

$$S_{\Delta ABC} = \frac{1}{2} h \times x + \frac{1}{2} h \times y = \frac{1}{2} h(x + y) = \frac{1}{2} h \times a$$



For an obtuse triangle it is not so obvious for the altitude drawn from the acute angle vertex.

Draw 3 arbitrary triangles. Draw three medians in the first triangle (use ruler for measuring), three altitude in the second triangle (use a triangular ruler), three angle bisectors in the third triangle (use protractor).

Exercises:

7. Draw two segments, the ratio of lengths of which is 2:3.
8. Draw a rectangle with the ration of sides 5:3
9. Draw a right triangle with the ratio of legs 3:4. Measure the length of the legs and hypotenuse and find the ratio of the length of each legs and hypotenuse.
10. Draw the angle of 60°. Divide the angle into two angles in ratio 1;2.