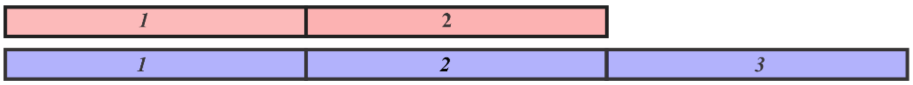
A and G 1. Class work 6..

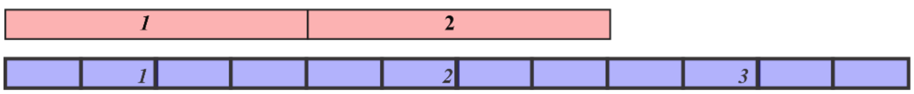
Algebra.

Step 1. For each 2 red balloons there are three blue balloons, so we can show all red and blue balloons as:

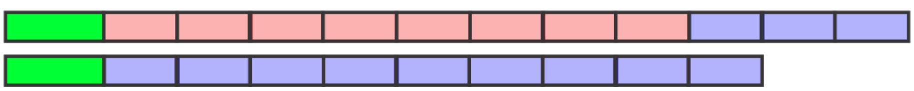


We took as "unit" a half of the red balloons. The number of blue balloons is $\frac{3}{2}$ times more.

Step 2. $\frac{3}{4}$ of the blue balloons were sold. We can't divide 3 "units" into 4 parts, without getting fractions. So, let's find LCM of 3 and 4 and divide the number of blue balloons into 12 parts.



Step 3. Let's compare the number of sold and leftover balloons.



922

$1686 - 922 = 764$

Number of sold and unsold green balloons are the same, red balloons are all left, as well as $\frac{1}{4}$ of blue balloons. As we can see 2 "units" of blue balloons are $922 - 764 = 158$, or one "unit" is 79. Total amount of blue balloons is $158 \cdot 6 = 948$. The number of red balloons is

$$\frac{2}{3} \cdot 948 = 632.$$

Number of red ones is $1686 - (632 + 948) = 106$

1 percent of quantity is a $\frac{1}{100}$ th part of it.

One percent (**1%**) means 1 per 100.

1% of this line is shaded green: it is very small isn't it?

In mathematics, a **ratio** is a relationship between two numbers indicating how many times the first number contains the second. For example, if a bowl of fruit contains eight oranges and six lemons, then the ratio of oranges to lemons is eight to six (that is, $8:6$, which is equivalent to the ratio $4:3$). Similarly, the ratio of lemons to oranges is $6:8$ (or $3:4$) and the ratio of oranges to the total amount of fruit is $8:14$ (or $4:7$).
The numbers in a ratio may be quantities of any kind, such as counts of persons or objects, or such as measurements of lengths, weights, time, etc. In most contexts both numbers are restricted to be positive, and the second to be not zero.
Ratios are sometimes used with three or even more terms, for example, a good concrete mix (in volume) is

$$cement:send:gravel \ = \ 1:2:4$$

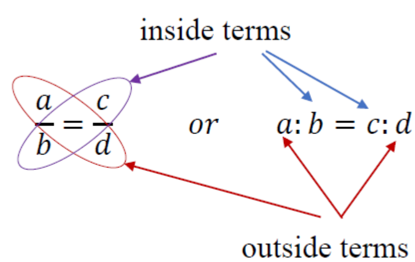
It means that for 1 volume part of cement go 2 volume parts of send and 4 volume parts of gravel.
In a nut mix the ratio of nuts is

$$walnats:almonds:haselnuts = 3:5:7$$

The bag of this mix weights 1500 g. What is the weight of almonds?
If a mixture contains substances A, B, C and D in the ratio $5:9:4:2$ then there are 5 parts of A for every 9 parts of B, 4 parts of C and 2 parts of D. As $5 + 9 + 4 + 2 = 20$, the total mixture contains $5/20$ of A (5 parts out of 20), $9/20$ of B, $4/20$ of C, and $2/20$ of D. If we divide all numbers by the total and multiply by 100, we have converted to percentages: 25% A, 45% B, 20% C, and 10% D (equivalent to writing the ratio as $25:45:20:10$).
To cook a raspberry jam according to recipe I need to combine three cups of berries and 2 cups of sugar, or for each 3 cups of raspberries go 2 cups of sugar; ratio of raspberries and sugar (in volume) is $3:2$. If I bought 27 cups of raspberries, how many cups of sugar do I need to put to my jam?

$$\frac{3}{2} = \frac{27}{x}$$

Two ratios which are equal form a proportion.
Proportions have several interesting features.



1. The product of inside and outside terms are equal.

$$\frac{a}{b} = \frac{c}{d} \quad \Leftrightarrow \quad a \cdot d = b \cdot c$$

It can be easily shown:

$$\frac{a}{b} = \frac{c}{d} \Leftrightarrow \frac{adb}{b} = \frac{cdb}{d} \Leftrightarrow ad = cb$$

2. Also, two inverse ratios are equal:

$$\frac{a}{b} = \frac{c}{d} \quad \Leftrightarrow \quad \frac{b}{a} = \frac{d}{c}$$

Indeed:

$$\frac{a}{b} = \frac{c}{d} \quad \Leftrightarrow \quad a \cdot d = b \cdot c \quad \Leftrightarrow \quad \frac{ad}{ac} = \frac{bc}{ac} \quad \Leftrightarrow \quad \frac{d}{c} = \frac{b}{a}$$

3. Two outside terms can be switched:

$$\frac{a}{b} = \frac{c}{d} \quad \Leftrightarrow \quad \frac{d}{b} = \frac{c}{a}$$

$$\frac{a}{b} = \frac{c}{d} \quad \Leftrightarrow \quad a \cdot d = b \cdot c \quad \Leftrightarrow \quad \frac{ad}{ab} = \frac{bc}{ab} \quad \Leftrightarrow \quad \frac{d}{c} = \frac{b}{a}$$

4. Two inside terms can be switched as well.

$$\frac{a}{b} = \frac{c}{d} \quad \Leftrightarrow \quad \frac{a}{c} = \frac{b}{d}$$

5. Also, several other new proportion can be created.

$$\frac{a}{b} = \frac{c}{d} \quad \Leftrightarrow \quad \frac{a \pm b}{b} = \frac{c \pm d}{d}$$

(the sign $\pm$ is used to show that both, addition and subtraction, can be used)

Let's prove one of the statements:

$$\frac{a}{b} = \frac{c}{d} \quad \Leftrightarrow \quad \frac{a}{b} + 1 = \frac{c}{d} + 1 \quad \Leftrightarrow \quad \frac{a}{b} + \frac{b}{b} = \frac{c}{d} + \frac{d}{d} \quad \Leftrightarrow \quad \frac{a+b}{b} = \frac{c+d}{d}$$

6. Another proportion:

$$\frac{a}{b} = \frac{c}{d} \quad \Leftrightarrow \quad \frac{a+c}{b+d} = \frac{c}{d} = \frac{a}{b}$$

It can be proved as follow:

$$\frac{a+c}{b+d} = \frac{a\left(1 + \frac{c}{a}\right)}{b\left(1 + \frac{d}{b}\right)}$$

We know form (4) that

$$\frac{c}{a} = \frac{d}{b}$$

$$\frac{a+c}{b+d} = \frac{a\left(1 + \frac{c}{a}\right)}{b\left(1 + \frac{d}{b}\right)} = \frac{d}{b}$$

Going back to the jam problem above. We got the simple equation

$$\frac{3}{2} = \frac{27}{x}$$

It can be solved easily using the property of proportion

$$3x = 27 \cdot 2$$
$$x = \frac{27 \cdot 2}{3} = \frac{3 \cdot 9 \cdot 2}{3} = 18$$

1. Three solutions of salt with concentration 10%, 15%, and 30% (it means that in the solution there are 10% (or 15%, or 30%) of the total mass is NaCl and 90% (or 85%, or 70%) is water) are mixed together. The mass of the first solution is 180g, mass of the second solution is twice as the mass of the first solution, and the mass of the third solution is 100 g. greater than the mass of the second solution. What is the concentration of the mixture?
2. Solve the following equations (hint: use the property of proportions):

   $a.\ \frac{x}{7.2} = \frac{1\frac{1}{9}}{0.25};\qquad b.\ \frac{2\frac{1}{3}}{0.6x} = \frac{2.5}{1\frac{2}{7}};\qquad c.\ \frac{\frac{7}{12}}{0.14} = \frac{50x}{4.8};\qquad d.\ \frac{1\frac{3}{17}}{13.75} = \frac{2\frac{2}{11}}{3x}$
3. Solve the following equations:
   - $a.\ x - 2\left(x - 3\left(x - 4(x - 5)\right)\right) = 6$
   - $b.\ 5x - 4(x - 3(x - 2(x - 1))) = 2$
   - $c.\ x - (x - (x - (x - 1))) = 1 - (2 - (3 - (4 - x)))$
   - $d.\ 4x - (3x - (2x - (x - 1) - 2) - 3) - 4 = 0$

4. In a department store, there is a sale of 25% off on everything. How much does the dress cost if its price before sale was $80? How much this dress will cost if an additional sale of 30% of will be applied?

5. There are 40000 books in a library. 75% of all books are in English, 10% of all books are in Spanish and the rest of the books are in French and German. How many books are there in the library in English and in Spanish?

6. Grapes were dried to raisin. During the process, the weight of grapes was reduced by 70%. How many kilograms of raisin was produced from 200 kg of grapes? How many kilograms of grapes were dried if the weight of obtained raisin is 15 kg?

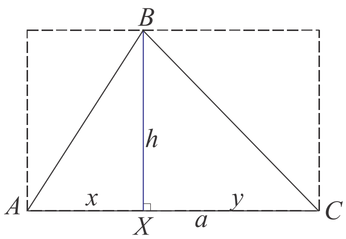
7. Dry cranberries contain 25% of water. How much water should be evaporated from 5 kg of fresh cranberries to get dry cranberries, if fresh cranberries contain 85% of water?

8. A book is 25% more expensive then a notebook. How many percent the notebook is less expensive then the book?

Geometry.

Area of a triangle.

The area of a triangle is equal to half of the product of its altitude and the base, corresponding to this altitude.

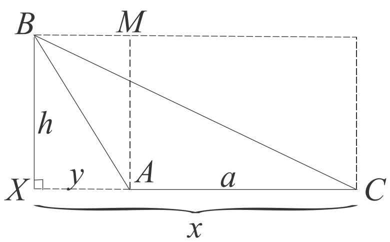


For the acute triangle it is easy to see.

$$S_{rec} = h \times a = x \times h + y \times h$$

$$S_{\Delta ABX} = \frac{1}{2} h \times x, \qquad S_{\Delta XBC} = \frac{1}{2} h \times y, \qquad S_{\Delta ABC} = S_{\Delta ABX} + S_{\Delta XBC}$$

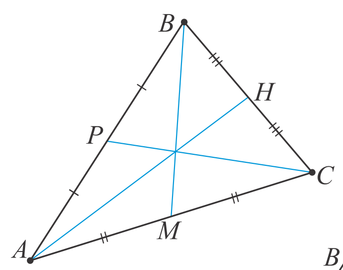
$$S_{\Delta ABC} = \frac{1}{2} h \times x + \frac{1}{2} h \times y = \frac{1}{2} h(x+y) = \frac{1}{2} h \times a$$



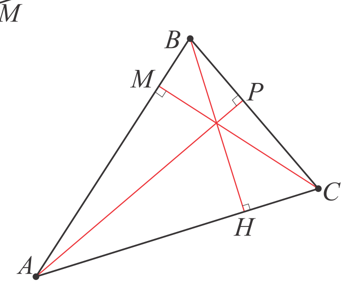
For an obtuse triangle it is not so obvious for the altitude drawn from the acute angle vertex.

Draw 3 arbitrary triangles. Draw three medians in the first triangle (use ruler for measuring), three altitude in the second triangle (use a triangular ruler), three angle bisectors in the third triangle (use protractor).

A **median** of a triangle is a line segment joining a vertex to the midpoint of the opposing side, bisecting it.



An **altitude** of a triangle is a line segment through a vertex and perpendicular to (i.e., forming a right angle with) a line containing the base (the side opposite the vertex).



A **bisector** of a triangle is a line segment through a vertex to the opposite side which cuts the corresponding angle in half.