

Algebra.

We already know what is GCD and LCM for several natural numbers and we know how to find them.

Exercise:

Find GCD (GCF) and LCM for numbers

- 222 and 345.
- $2^2 \cdot 3^3 \cdot 5$ and $2 \cdot 3^2 \cdot 5^2$

Can we apply the same strategy to find CF and CM for algebraic expressions? (In this case the concept of GCD and LCM cannot be applied.) For example, can CF and CM be found for expressions $2x^2y^5$ and $4x^3y^2$? x and y are variables and can't be represented as a product of factors, but they itself are factors, and the expression can be represented as a product:

$$2x^2y^5 = 2 \cdot x \cdot x \cdot y \cdot y \cdot y \cdot y \cdot y,$$

$$4x^3y^2 = 2 \cdot 2 \cdot x \cdot x \cdot x \cdot y \cdot y.$$

$$A = (\text{Factors}, 2x^2y^5) = \{2, x, x, y, y, y, y, y\}, B = (\text{Factors}, 4x^3y^2) = \{2, 2, x, x, x, y, y\}$$

Common devisors are any product of $A \cap B = \{2, x, x, y, y\}$.

What about common multiples? Product of all factors of both numbers (or the product of two numbers) will be the multiple, but minimal common multiple will be the product of the

$$A \cup B = \{2, 2, x, x, x, y, y, y, y, y\}$$

$$\frac{2x^2y^5}{2 \cdot x^2y^2} = y^3; \quad \frac{4x^3y^2}{2 \cdot x^2y^2} = 2x;$$

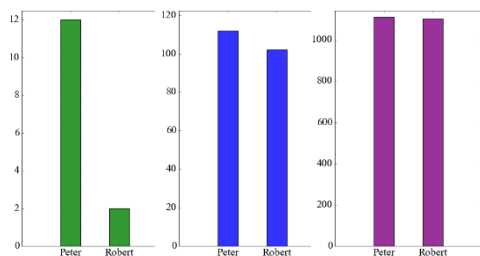
$$\frac{2x^3y^5}{2 \cdot x^2y^5} = x; \quad \frac{4x^3y^5}{4 \cdot x^3y^3} = y^2; \quad x, y \neq 0$$

Ratio and percent.

Peter has 10 dollars more then Robert. Is this a big difference? How we can compare the amount of money they have?

Take a look at the table

Peter	\$12	\$112	\$1112
Robert	\$2	\$101	\$1102



In all these cases the absolute difference is the same, but in the first situation Peter has 6 times as much as Robert, in the last situation they both have almost the same amount of money. The ratios of the amount of Peter's money and Robert's money are

$$\frac{12}{2}; \quad \frac{112}{102}; \quad \frac{1112}{1102};$$

The quantitative relation between two amounts showing the number of times one value contains or is contained within the other is called a ratio. The amount of money Peter and Robert have in the first case is 12 and 2 dollars and the ratio is $\frac{12}{2} = 6$, or 6:1, or 6 to 1.

The ratio of two numbers indicates how many times one number is larger than another or which part of one number the other number is.

(it's not a real recipe) The ratio of water and lemon juice in lemonade is 4 to 1. What does it mean? It means that for each part of lemon juice we need to add 4 parts of water (it's not a real recipe).

How much of lemon juice and water we need to prepare 1 l. of lemonade? 1.5 l.?

We want to have sweet lemonade and we add sugar. The ratio of water, lemon juice and sugar is 4:1:0.5 (or it can be rephrased as 8:2:1). For each part of sugar, we will use 2 parts of lemon juice, and 8 parts of water.

Three brothers, 5, 7, and 9 years old went to trick-o-treat. They got 84 sweets altogether. They decided to divide the candies in the ratio of their age 9:7:5. How many candies each of should get?

To divide all candies between the brothers we need to find the "unit" part of the total amount of candies. The oldest brother should get 9 of such units, the middle one should get 7, and the youngest brother will get 5. Total amount of units is $9 + 7 + 5 = 21$. The number of candies is 84, so the "unit" contains $84 : 21 = 4$ candies.

So, the first brother will get $4 \cdot 9 = 36$ candies, the second will get $4 \cdot 7 = 28$, and the third will get $4 \cdot 5 = 20$. $36 + 28 + 20 = 84$

We can write the ratio of two numbers in the several ways:

$$a \text{ to } b, \quad a:b, \quad \frac{a}{b}$$

Irene has a total of 1686 red, blue and green balloons for sale. The ratio of the number of red balloons to the number of blue balloons was 2:3. After Irene sold $\frac{3}{4}$ of the blue balloons, $\frac{1}{2}$ of the green balloons and none of the red balloons, she has 922 balloons left. How many blue balloons did Irene have at first?

Step 1. For each 2 red balloons there are three blue balloons, so we can show all red and blue balloons as:



We took as “unit” a half of the red balloons. The number of blue balloons is $\frac{3}{2}$ times more.

Step 2. $\frac{3}{4}$ of the blue balloons were sold. We can’t divide 3 “units” into 4 parts, without getting fractions. So, let’s find LCM of 3 and 4 and divide the number of blue balloons into 12 parts.



Step 3. Let’s compare the number of sold and leftover balloons.



Number of sold and unsold green balloons are the same, red balloons are all left, as well as $\frac{1}{4}$ of blue balloons. As we can see 2 “units” of blue balloons are $922 - 764 = 158$, or one “unit” is 79. Total amount of blue balloons is $158 \cdot 6 = 948$. The number of red balloons is

$$\frac{2}{3} \cdot 948 = 632.$$

Number of red ones is $1686 - (632 + 948) = 106$

Can we solve the problem by writing equations?

Let’s try.

$$G + B + R = 1686$$

$$3R = 2B$$

$$\frac{1}{2}G + R + \frac{1}{4}B = 922$$

$$\frac{1}{2}G + R + \frac{1}{4}B - \left(\frac{1}{2}G + \frac{3}{4}B\right) = 922 - 764$$

$$R - \frac{1}{2}B = 158$$

$$\frac{2}{3}B - \frac{1}{2}B = 158 \Rightarrow \left(\frac{4}{6} - \frac{3}{6}\right)B = 79 \Rightarrow B = 6 \cdot 158$$

1 percent of quantity is a $\frac{1}{100}$ th part of it.

1. One percent (1%) means 1 per 100.



1% of this line is shaded green: it is very small isn't it?

2. How many squares we have to shade to shade 10% of the line, 15%, 20%, 25%?
3. In a department store, there is a sale of 25% off on everything. How much does the dress cost if its price before sale was \$80? How much this dress will cost if an additional sale of 30% of will be applied?
4. There are 40000 books in a library. 75% of all books are in English, 10% of all books are in Spanish and the rest of the books are in French and German. How many books are there in the library in English and in Spanish?
5. Grapes were dried to raisin. During the process, the weight of grapes was reduced by 70%. How many kilograms of raisin was produced from 200 kg of grapes? How many kilograms of grapes were dried if the weight of obtained raisin is 15 kg?
6. In a dried fruit mix, there are 7parts of dried apples, 4 parts of dried pears and 5 parts of dried apricots. What is the weight (how many grams) of apples, pears, and apricots in the fruit mix, if the total weight of the mix is 1600g?
7. In order to prepare a homemade dried fruits and nuts mix Mary took 6 parts of raisins, 5 parts of dried cranberries and 3 parts of walnuts. Cranberries and walnuts altogether weighted 2 kg 400 g. What was the weight of the mix that Mary prepared?
8. To do her homework, Julia solved math problems, wrote an essay, and did a history project. It took her 2 hours and 15 minutes to finish all the assignments. The ratios of the

times she spends doing math, writing the essay, and doing history project are 3:2:1. How much time did she spend for each of her subjects?

9. A book is 25% more expensive than a notebook. How many percent the notebook is less expensive than the book?

Famous ratios.

- i. Let's measure the circumference and the diameter of a circle.

$$\frac{l}{d} = \pi$$

- ii. Golden ratio:

$$\frac{a+b}{a} = \frac{a}{b} \cong 1.62$$

