A and G1. Class work 5.

Algebra.

Positive rational number is a number which can be represented as a ratio of two natural numbers:

$$a = \frac{p}{q}; \qquad p, q \ \in N$$

As we know such number is also called a fraction, p in this fraction is a nominator and q is a denominator. Any natural number can be represented as a fraction with denominator 1:

$$b = \frac{b}{1}; \ \ b \in N$$

Basic property of fraction: nominator and denominator of the fraction can be multiplied by any non-zero number n, resulting the same fraction:

$$a = \frac{p}{q} = \frac{p \cdot n}{q \cdot n}$$

In the case that numbers p and q do not have common prime factors, the fraction $\frac{p}{q}$ is irreducible fraction. If $p < q$, the fraction is called “proper fraction”, if $p > q$, the fraction is called “improper fraction”.

If the denominator of fraction is a power of 10, this fraction can be represented as a finite decimal, for example,

$$\frac{37}{100} = \frac{37}{10^2} = 0.37, \qquad \frac{3}{10} = \frac{3}{10^1} = 0.3, \qquad \frac{12437}{1000} = \frac{12437}{10^3} = 12{,}437$$

$$10^n = (2 \cdot 5)^n = 2^n \cdot 5^n$$

$$\frac{2}{5} = \frac{2}{5^1} = \frac{2 \cdot 2^1}{5^1 \cdot 2^1} = \frac{4}{10} = 0.4$$

$$\begin{array}{r|l} & \ \ 0.875 \\ \hline 8 & 7.000 \\ & -6.4 \\ & \quad\ 60 \\ & \ \ -56 \\ & \qquad 40 \\ & \quad\ -40 \\ & \qquad\ 0 \end{array}$$

Therefore, any fraction, which denominator is represented by $2^n \cdot 5^m$ can be written in a form of finite decimal. This fact can be verified with the help of the long division, for example $\frac{7}{8}$ is a proper fraction, using the long division this fraction can be written as a decimal $\frac{7}{8} = 0.875$. Indeed,

$$\frac{7}{8} = \frac{7}{2 \cdot 2 \cdot 2} = \frac{7 \cdot 5 \cdot 5 \cdot 5}{2 \cdot 2 \cdot 2 \cdot 5 \cdot 5 \cdot 5} = \frac{7 \cdot 5^3}{2^3 5^3} = \frac{7 \cdot 125}{(2 \cdot 5)^3} = \frac{875}{10^3} = \frac{875}{1000} = 0.875$$

Also, any finite decimal can be represented as a fraction with denominator 10^n.

$$0.375 = \frac{375}{1000} = \frac{3}{8} = \frac{3}{2^3}; \qquad 0.065 = \frac{65}{1000} = \frac{13 \cdot 5}{5^3 2^3} = \frac{13}{5^2 2^3};$$

$$6.72 = \frac{672}{100} = \frac{168}{25} = \frac{168}{5^2}; \qquad 0.034 = \frac{34}{1000} = \frac{17 \cdot 2}{5^3 2^3} = \frac{17}{5^3 2^2};$$

$$\begin{array}{r@{}l}
 & 0.71428571\ldots \\
7 & \overline{)5.000} \\
-0\,0 & \\
\hline
5\,0 & \\
-4\,9 & \\
\hline
10 & \\
-\ 7 & \\
\hline
30 & \\
-28 & \\
\hline
20 & \\
-14 & \\
\hline
60 & \\
-56 & \\
\hline
40 & \\
-35 & \\
\hline
50 & \\
-49 & \\
\hline
10 & \\
\ldots\ldots &
\end{array}$$

In other words, if the finite decimal can be represented as an irreducible fraction, the denominator of this fraction will not have other factors besides 5^m and 2^n. Converse statement is also true: if the irreducible fraction has denominator which only contains 5^m and 2^n than the fraction can be written as a finite decimal. (Irreducible fraction can be represented as a finite decimal if and only if it has denominator containing only 5^m and 2^n as factors.)

If the denominator of the irreducible fraction has a factor different from 2 and 5, the fraction cannot be represented as a finite decimal. If we try to use the long division process, we will get an infinite periodic decimal.

At each step during this division we will have a remainder. At some point during the process we will see the remainder which occurred before. Process will start to repeat itself. On the example on the left, $\frac{5}{7}$, after 7, 1, 4, 2, 8, 5, remainder 7 appeared again, the fraction $\frac{5}{7}$ can be represented only as an infinite periodic decimal and should be written as $\frac{5}{7} = 0.\overline{714285}$. (Sometimes you cen find the periodic infinite decimal written as $0.\overline{714285} = 0.(714285)$).

How we can represent the periodic decimal as a fraction?

Let's take a look on a few examples: $0.\overline{8}$, $2.35\overline{7}$, $0.\overline{0108}$.

1. $0.\overline{8}$.

$x = 0.\overline{8}$

$10x = 8.\overline{8}$

$10x - x = 8.\overline{8} - 0.\overline{8} = 8$

$9x = 8$

$x = \frac{8}{9}$

2. $2.35\overline{7}$

$x = 2.35\overline{7}$

$100x = 235.\overline{7}$

$1000x = 2357.\overline{7}$

$1000x - 100x = 2357.\overline{7} - 235.\overline{7}$
$= 2122$

$x = \frac{2122}{900} = \frac{1061}{450}$

3. $0.\overline{0108}$

$x = 0.\overline{0108}$

$10000x = 108.\overline{0108}$

$10000x - x = 108$

$x = \frac{108}{9999} = \frac{12}{1111}$

Now consider $2.4\overline{0}$ and $2.3\overline{9}$

$x = 2.4\overline{0}$

$10x = 24.\overline{0}$

$100x = 240.\overline{0}$

$100x - 10x = 240 - 24$

$x = \frac{240 - 24}{90} = \frac{216}{90} = 2.4$

$x = 2.3\overline{9}$

$10x = 23.\overline{9}$

$100x = 239.\overline{9}$

$100x - 10x = 239 - 23$

$x = \frac{239 - 23}{90} = \frac{216}{90} = 2.4$

Exercises.

1. Represent the following fractions as decimals:

$a.\ \frac{3}{2000},$ $b.\ \frac{17}{40};$ $c.\ \frac{28}{140};$

$d.\ \frac{7}{4};$ $e.\ \frac{3}{2};$ $f.\ \frac{9}{5};$

$g.\ \frac{123}{20};$ $h.\ \frac{783}{540};$ $i.\ \frac{324}{25};$

2. Write as a fraction:

$a.\ 0.\overline{3},$ $b.\ 0.3,$ $c.\ 0.\overline{7},$ $d.\ 0.7,$

$e.\ 0.1\overline{2},$ $f.\ 0.\overline{12},$ $g.\ 0.12,$ $h.\ 1.12\overline{3},$

$i.\ 7.5\overline{4},$ $j.\ 1.0\overline{12}.$

3. Compute:

a. $(-3)^3;$
b. $-3^3;$
c. $(-3)^4$
d. -3^3
e. $-2^7;$
f. $(-2)^7;$
g. $(2\cdot 3)^3;$
h. $2\cdot 3^3;$
i. $\left(\frac{1}{3}\right)^2;$
j. $\frac{1}{3^2};$
k. $3^{-2};$
l. $(-3)^{-2};$
m. $(-5\cdot 2)^3$

Remember, that $a^n : a^m = a^{n-m} = a^{n+(-m)} = a^n \cdot \frac{1}{a^m} = a^n \cdot a^{-m}$

4. Sum of two natural numberы is 45. First number will give the remainder 4 upon division by 12, second number will give the remainder 5 upon division by 12. What are these numbers?

5. Compare (replace … with >, <, or =) if possible, if it is known that *a* and *b* are positive numbers and *x* and *y* are negative numbers:

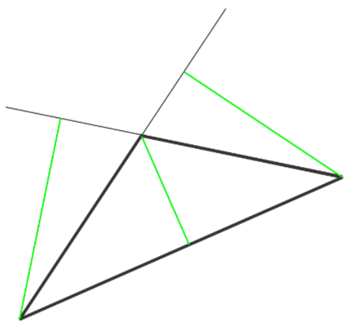
$0\ \dots\ x$	$a\ \dots\ 0$	$-b\ \dots\ 0$	$0\ \dots -x$
$a\ \dots\ x$	$y\ \dots\ b$	$-y\ \dots\ x$	$-a\ \dots\ b$
$\lvert x\rvert\ \dots\ x$	$-\lvert y\rvert\ \dots\ y$	$a\ \dots\ \lvert a\rvert$	$\lvert b\rvert\ \dots\ \lvert -b\rvert$
$\lvert x\rvert\ \dots\ a$	$\lvert x\rvert\ \dots -x$	$\lvert x\rvert\ \dots -\lvert y\rvert$	$a\ \dots\ \lvert -b\rvert$

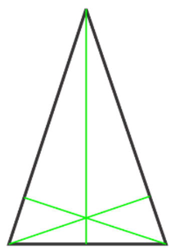
6. On the island of knights and knaves, you meet two inhabitants: Zoey and Mel. Zoey tells you that Mel is a knave. Mel says, “Neither Zoey nor I are knaves.” So, who is a knight and who is a knave? (Knights always tell the truth, and knaves always lie).

Geometry.

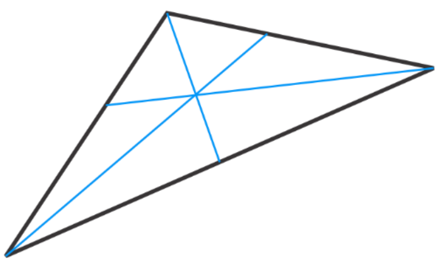
Special segments of a triangle.

From each vertices of a tringle to the opposite side 3 special segment can be constructed.





An **altitude** of a triangle is a straight line through a vertex and perpendicular to (i.e. forming a right angle with) the opposite side. This opposite side is called the *base* of the altitude, and the point where the altitude intersects the base (or its extension) is called the *foot* of the altitude.



An **angle bisector** of a triangle is a straight line through a vertex which cuts the corresponding angle in half.

A **median** of a triangle is a straight line through a vertex and the midpoint of the opposite side, and divides the triangle into two equal areas.