A and G1. Class work 3.

Algebra.

Factorization.

In mathematics factorization is a decomposition of one number into a product of two or more numbers (or representation of an expression as a product of 2 or more expressions) which are called factors. For example, we can represent the expression $a \cdot b + a \cdot c$ as a product of a and expression $(b + c)$. Can you explain why?

$$a \cdot b + a \cdot c = a \cdot (b + c)$$

Can an even number be a prime number? Is there any even prime number?

Prime factorization or integer **factorization** of a number is the determination of the set of **prime** numbers which multiply together to give the original integer. It is also known as **prime** decomposition.

fundamental theorem of arithmetic:

Any natural number greater than 1 either is a prime number or can be represented as a product of prime numbers and such representation is unique.

For example:

$$1200 = 5 \cdot 2 \cdot 5 \cdot 2 \cdot 3 \cdot 2 \cdot 2$$

The theorem says two things, for this example: first, that 1200 can be represented as a product of primes, and second, that no matter how this is done, there will always be exactly four 2s, one 3, two 5s, and no other primes in the product. This theorem is a main reason why 1 is not considered a prim number.

LCM and GCF (GCD).

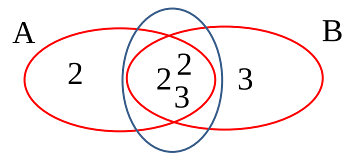
Each natural number is a prime number or can be represented as product of a unique set of prime numbers (see above, fundamental theorem of arithmetic). How we can find all divisors of a number? If the number is prime, there is no other divisors, but itself. If the number is not prime – each prime factor is a divisor, as well as product of all possible combinations (subsets of the set of prime factors). For example, number 24 has a prime representation $24 = 2 \cdot 2 \cdot 2 \cdot 3$, therefore 4, 8, 6, and 12, as well as 24, will be also the divisors of 24. Any two (or more) natural numbers can have common divisors (such that both numbers can be divided by evenly), or, in case that there are no such common divisors, they are called mutually prime. For example, 9 and 20:

$$9 = 3 \cdot 3, \qquad 20 = 2 \cdot 2 \cdot 5$$

These two numbers are not prime, but because they don’t have common divisors, they are mutually prime.
How to find common factor? Take a look at the prime factorization of numbers 24 and 36:

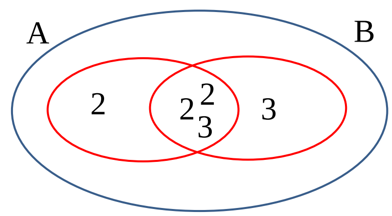
$$24 = 2 \cdot 2 \cdot 2 \cdot 3$$
$$36 = 2 \cdot 2 \cdot 3 \cdot 3$$

Both numbers have common factors, so they both can be divided two times by 2 and by 3, and by the product of any combinations of these three numbers. The greatest divisor (greatest common factor will be the product of all common factors. This can be also be represented as the Venn diagrams of the sets of prime factors of numbers 24, 36, and the intersection of these two sets. Set A = $(P, 24)$,   B = $(P, 36)$



Multiple of a number a is any number, which is divisible by a. $(M = na,\ n \in N)$. For two numbers, a and b common multiple is a number which is divisible by both, a and b. One of the common multiples is the product of a and b, but it’s not necessarily the smallest one.

$$24 = 2 \cdot 2 \cdot 2 \cdot 3$$
$$36 = \ \ 2 \cdot 2 \cdot 3 \cdot 3$$



The product of the common prime factors together with the remaining prime factors from both numbers will be divisible by both numbers, and will be the smallest multiple. In terms of set theory, this will be the product of all elements of the unity of sets A an B.

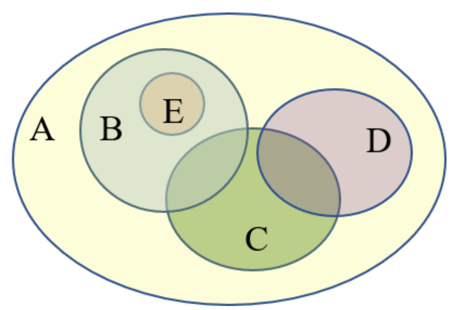
Exercises:

1. Among four consecutive natural numbers will be at least one even number? At least one number divisible by 3? By 4? By 5? Explain your answer.
2. Can the expression below be a true statement, if the letters are replaced with numbers from 1 to 9 (different letters represent different numbers). Explain your answer.
$$f \cdot l \cdot y = i \cdot n \cdot s \cdot e \cdot c \cdot t$$
3. M is a set of prime factors of number 28, K is a set of prime factors of number 36. Draw the Venn diagrams of two sets, find the intersection and union of the sets. Write them in a form M∩N = {...} and M∪N = {...}.
What are the divisors of 28 and 36?
4. Will the quotient and remainder change if dividend and divisor are increased 3 times?
5. Two buses leave from the same bus station following two different routes. For the first one it takes 48 minutes to complete the roundtrip route. For the second one it takes 1 hour and 12 minutes to complete the round trip route. How much time will it take for the buses to meet at the bus station for the first time after the have departed for their routes at the same time?
6. A florist has 36 roses, 90 lilies, and 60 daisies. What is largest amount of bouquets the florist can create from these flowers evenly dividing each kind of flowers between them?

7. A, B, C and D are coordinates of four points on a number line below. Which coordinates are multiple of *n*? of *m*? which are common multiple of *n* and *m*, which is LSM? (use compass to measure distances).

8. There are 4 stories in the book. The first story is 12 pages long, which is $\frac{2}{3}$ of the second story. Third story is $\frac{5}{6}$ of the sum of the first two stories. Which part of the book is the fourth story if there are 64 pages in the book?
9. Mary is reading a book 100 pages long. She read x pages in the morning, y pages in the afternoon. How many pages is left for her to read? $(x + y < 100)$ Write an expression with variables. Evaluate the expression if
   a. $x = 23, y = 25$
   b. $x = 45, y = 33$

10. Draw the Venn diagrams for the following sets and write the relationship for them. Write some examples of the members of each set:



Example:
Set A is a set of all animals, set B is a set of mammals, set C is a set of the animals that can fly, set D is the set of birds, set E is a set of wolfs.
$B\subset A, E\subset B,\ \ E\subset A, C\subset A, D\subset A$

$A\ woodpecker \in D,\quad A\ woodpecker \in C,\quad A\ woodpecker \in C \cap D$

$A\ bat\ \ \in B, a\ bat\ \in C, a\ bat\ \in C \cap B$

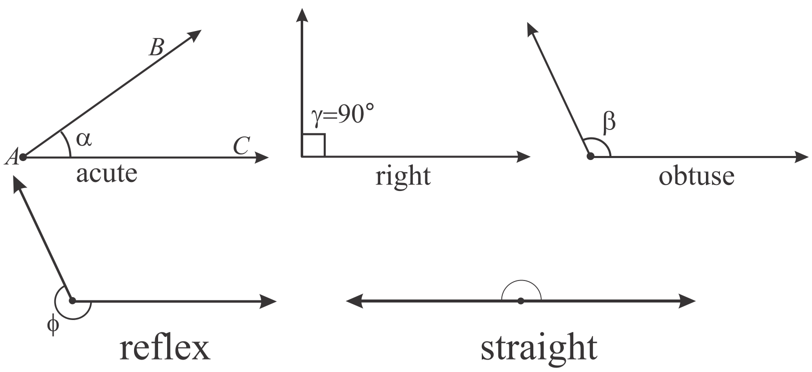
$A\ bee \in C$

   a. N is a set of natural numbers. K is a set of natural numbers greater than 10, P is a set of natural numbers smaller than 25. L is the set of natural numbers smaller than 18. $H = \left\{\frac{1}{2}, \frac{5}{2}, \frac{7}{3}\right\}$.
   b. M is a set of quadrilaterals. S is a set of squares, R is a set of rectangles, H is a set of rhombuses, T is a set of triangles.
   c. A is a set of all cities of the United States, B is a set of the capitals of the all states, set C contains one element, “Albany”. Set D = {Paris (France), Berlin (Germany), New Delhi (India), Moscow (Russia)}

Geometry.

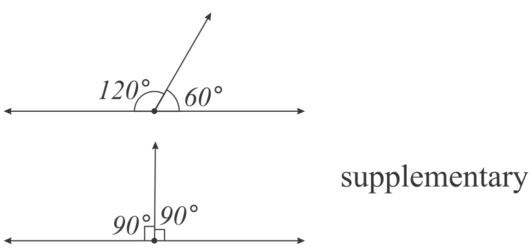
An angle is the figure formed by two **rays**, called the sides of the angle, sharing a common endpoint, called the **vertex** of the angle.

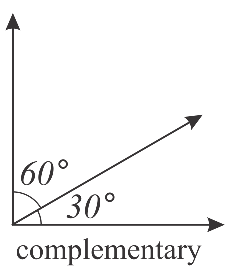
Angles notations are usually three capital letters with vertex letter in the middle or small Greek letter: $\angle ABC$, α. Measure of the angle is the amount of rotation required to move one side of the angle onto the other. As the angle increases, the name changes:



Strait angle is formed by two rays on the same strait line. Straight angle has a measure of 180°.

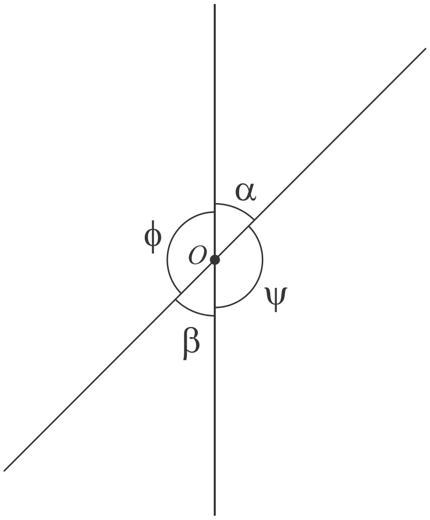
Two angles are called adjacent if they have common vertex and a common side. If two adjacent angles combined form straight angle they are called supplementary; if they form right angle than they are called complementary.





An angle which is supplementary to itself we call right angle. Lines which intersect with the right angle we call perpendicular to each other.

When two straight lines intersect at a point, four angles are formed. A pair of angles opposite each other formed by two intersecting straight lines that form an "X"-like shape, are called vertical angles, or opposite angles, or vertically opposite angles.



α and β and ϕ and ψ are 2 pairs of vertical angles.

Vertical angles theorem*:*

Vertical angles are equal.

In mathematics, a **theorem** is a statement that has been proven on the basis of previously established statements. According to a historical legend, when Thales visited Egypt, he observed that whenever the Egyptians drew two intersecting lines, they would measure the vertical angles to make sure that they were equal. Thales concluded that one could prove that vertical angles are always equal and there is no need to measure them every time.

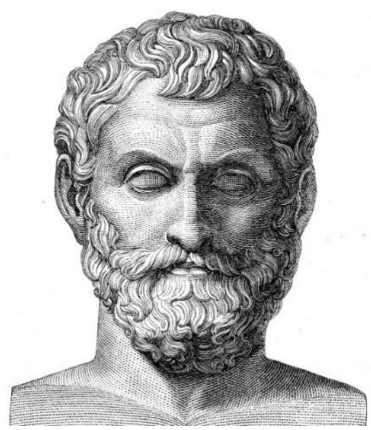
Proof:

$\angle\phi + \angle\alpha = 180°$ because they are supplementary by construction.

$\angle\phi + \angle\beta = 180°$ because they are supplementary also by construction.

$\Rightarrow \angle\alpha = \angle\beta$, therefore we proved that if 2 angles are vertical angles then they are equal. Can we tell that invers is also the truth? Can we tell that if 2 angles are equal than they are vertical angels?

(**Thales of Miletus** 624-546 BC was a Greek philosopher and mathematician from Miletus. Thales attempted to explain natural phenomena without reference to mythology. Thales used geometry to calculate the heights of pyramids and the distance of ships from the shore. He is the first known individual to use deductive reasoning applied to geometry, he also has been credited with the discovery of five theorems. He is the first known individual to whom a mathematical discovery has been attributed (Thales theorem).



Exercises.

11. Draw 3 different angles, measure them with a protractor.
12. Draw angles with the measures $30°, 72°, 45°, 135°, 153°, 90°$. Use ruler and protractor.
13. On a segment [AB] points P and M are marked. Point M lies between A and P, point P is a center of the segment [BM]. Draw a picture, find the length of the segment [AM], if the length of [AP]=6cm, [BP]=5 cm.
14. 4 angles are formed at the intersection of 2 lines. One of them is 30°. What is the measure of 3 others?
15. Draw 2 angles in such way that they intersect

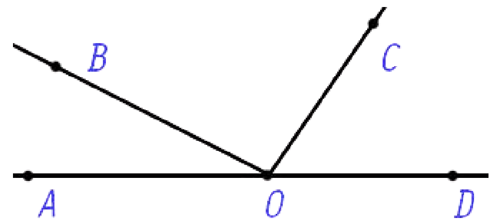
a. by a point

b. by a segment

c. by a ray

d. don't intersect at all.

16. 3 lines intersect at 1 point and form 6 angles. One is 44°, another is 38°. Can you find all other angles? Draw the picture. Use protractor and ruler.
17. Right angle is divided into 3 angles by 2 rays. One angle by 20° more than the other and by 20° less the third one. What are the measures of these 3 angles?
18. On the picture below ∠BOD = 152°, ∠COD = 55°, angle ∠AOD is a straight angle. Find the measures of all other angles on the picture.



19. On a line mark two points. How many segments were formed? Add one point. How many segments are there now? Add one more point. How many segments are there now? How many segments 6 points will form on the line? 10? 99?
20. On a line *l* there is a segment [AB]. 45 different points marked on the line outside if the segment [AB]. Can the sum of all distances from all 45 points to the point A be equal to the sum of the distances from these points to the point B?
21. The length of the segment [MN] = 12 cm. Point B divides the segment [MN] into two segments. Find the distance between the centers of these two segments. Draw a diagram.