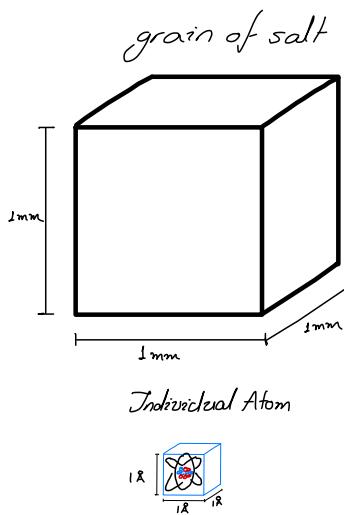


Length scales of the Universe

P1 Solution

We want to estimate how many atoms would be in a grain of salt whose volume is given by $1\text{mm} \times 1\text{mm} \times 1\text{mm}$.

Since we want an estimate, we can make some assumptions even if they are not very precise. Namely, you can assume an atom to be a cube as well. The problem now turns into finding how many small cubes (atoms) we can fit into a large cube (grain of salt).



Here is where physics helps us to answer this question. If the large cube is completely filled by N atoms, then the volume of both has to be the same. This allows us to write the following equation:

$$\underbrace{V_{\text{grain}}}_{\substack{\text{volume of} \\ \text{the salt grain}}} = \underbrace{N}_{\substack{\text{Number} \\ \text{of atoms}}} \underbrace{V_{\text{atom}}}_{\substack{\text{Volume} \\ \text{of an individual atom}}}$$

Then, the number of atoms in a grain of salt would be

$$N = \frac{V_{\text{grain}}}{V_{\text{atom}}}$$

To find the explicit result we need to substitute the value of each volume. From what we saw in class, you can approximate the size of an atom to be $1\text{\AA} = 10^{-10}\text{m}$. Then,

$$V_{\text{grain}} = 1\text{mm} \times 1\text{mm} \times 1\text{mm} = (10^{-3}\text{m}) \times (10^{-3}\text{m}) \times (10^{-3}\text{m}) = 10^{-9}\text{m}^3$$

$$V_{\text{atom}} = 1\text{\AA} \times 1\text{\AA} \times 1\text{\AA} = (10^{-10}\text{m}) \times (10^{-10}\text{m}) \times (10^{-10}\text{m}) = 10^{-30}\text{m}^3$$

So the number of atoms we can find in a grain of salt is

$$N = \frac{10^{-9}\text{m}^3}{10^{-30}\text{m}^3} = \frac{10^{-9}}{10^{-30}} \cdot \frac{10^{30}}{10^{30}} = 10^{21}$$

