2022 MATH 10 PLACEMENT EXAM

1. Compute the following expressions, writing them in the usual form $a + bi$:

$$\frac{1 - 2i}{3 + 4i}$$

$$(1 + i\sqrt{3})^{12}$$

2. Factor the following polynomial:

$$p(x) = x^3 - 7x - 6$$

(Hint: can you find at least one root?)

3. Consider the sequence defined by the following rules:

$$a_1 = 1, \quad a_{n+1} = 2a_n + 1$$

(a) Compute first 6 terms of this sequence. Do you see a pattern? can you guess a formula for a_n?

(b) Use mathematical induction to prove the formula you guessed in the previous part.

4. At a certain Pacific island, seashells and banana leaves are traditionally used as money, but now dollars are also accepted from tourists. One coconut costs a banana leaf and three seashells, while a smoked fish costs two banana leaves and two seashells. Knowing that a tourist will pay $3.50 for a coconut and $5.00 for a smoked fish, how much are banana leaves and seashells worth in dollars?