

Summer 2021, Intuitive Geometry.

Lesson 1

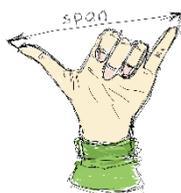
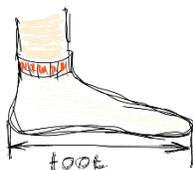
The word “geometry” is traced back to the Ancient Greece, “γεωμετρία”, geo means “earth”, “metron” means “measurement”. But the geometry itself is even older, people always were in need to use known facts about length, angles, areas in their life, for surveying (measuring) land, construction, astronomy, craft. The earliest known written texts about geometrical facts are the Egyptian Rhind Papyrus (2000–1800 BC) and the Babylonian clay tablets such as Plimpton 322 (1900 BC).

Later, in 7th century BC, the Greek mathematician Thales of Miletus used geometry to solve problems such as calculating the height of pyramids and the distance of ships from the shore. He first used the deductive reasoning applied to geometry, introduced the concept of proving the statement.

- *Why did they need the geometrical knowledge for astronomy?*
- *What are we doing when we are measuring something?*

We are comparing our object to be measured with something considered to be some sort of standard. It can be body part, it's always present, and easy to use.

- *What is not good about the measuring things with such units?*

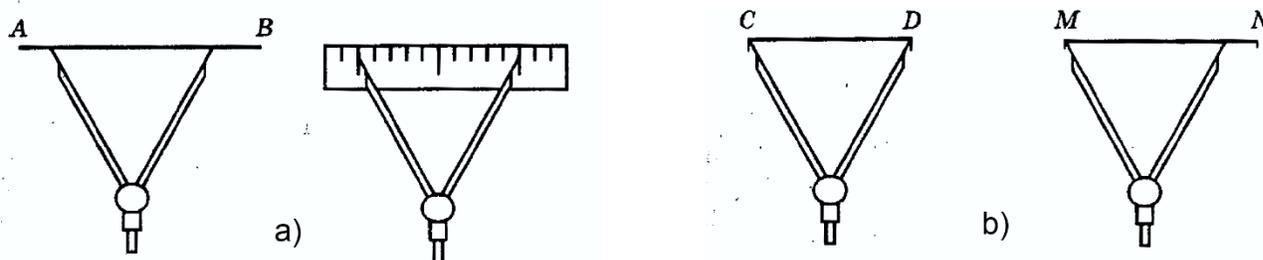


- *What other units of measurement do you know? What can be measured with them?*

Problem 1: How can we find the weight of one grain of rice if we have a scale and weights of 1g (3g, 5g)? How it can be done on the most accurate way?

In geometry class we will use the geometrical tools: ruler, compass and protractor. With ruler we can measure distances between two points, ruler without marks is called a

straight edge ruler, we can only draw straight lines with it. (In reality, the normal ruler is used, of course.) With compass we can draw a circle, also we can measure the length of a segment, compare the length of segments (a and b on the picture below):



- *What else are we usually measuring in geometry? Which tool do we need to use?*

Take a ruler and draw a segment. Your ruler has two scales, in centimeters and in inches. Measure your segment in inches and in centimeters. For example:



We got two different numbers, the number of inches and the of centimeters in the segment; length of the segment measured in two different measuring systems.

Problem 2: The length of one side of the rectangle is 2 inches, the length of the other side is 6 cm. What is the perimeter of this rectangle (perimeter is a sum of the length of all sides of the polygon)?

Problem 3: How to measure the weight of one grain of rice with the scale and weights of 1g, 2g, 3g. How can it be done most accurately?

A few lines above I gave the definition of “perimeter”. To clearly understand the idea, we need to know what is polygon (assuming that the meanings of all other words is identified). Then we can say that

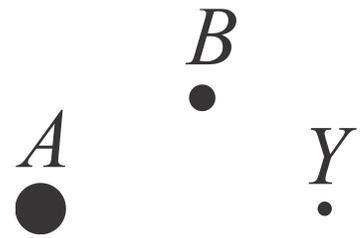
a polygon is a closed area of the plane bounded by a set of segments, in a such way that the end of a one segment is a beginning of the next, and the last segment ends at the beginning of the first one.

The definition of the polygon is a little bit long and includes a few new words, such as a segment, plane, closed area. Now we need to define what a segment, a plane, a closed area is? And so on. Mathematicians decided to have a few concepts which are intuitively clear and don't need to be clearly defined. These concepts are used as building blocks to describe all other geometrical ideas.

Mathematicians decided do not define terms "point", "straight line", and "plane" and to rely upon intuitive understanding of these terms.

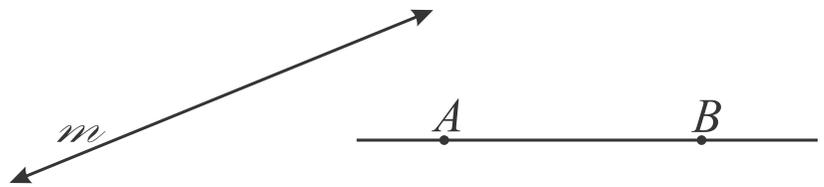
Point (an undefined term).

In geometry, a point has no dimension (actual size), point is an exact location in space. Although we represent a point with a dot, the point has no length, width, or thickness. Our dot can be very tiny or very large and it still represents a point. A point is usually named with a capital letter.



Line (an undefined term).

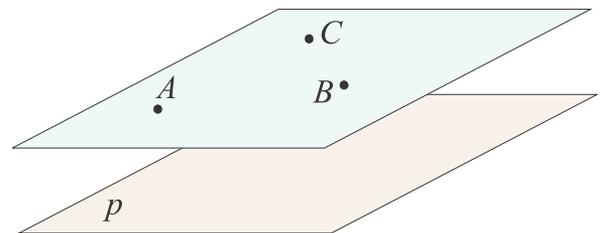
In geometry, a line has no thickness but its length extends in one dimension and goes on forever in both directions. Unless



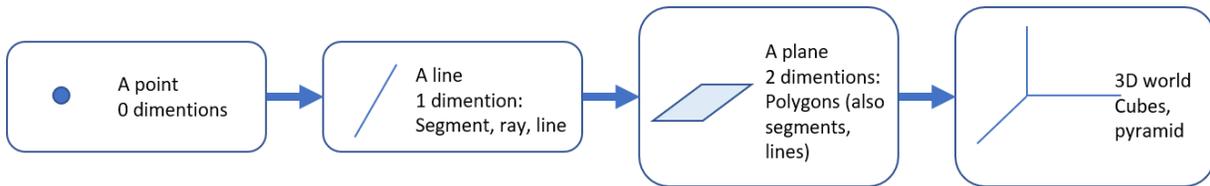
otherwise stated a line is drawn as a straight line with two arrowheads indicating that the line extends without end in both directions (or without them). A line is named by a single lowercase letter, m for example, or by any two points on the line, (\overline{AB}) or AB .

Plane (an undefined term).

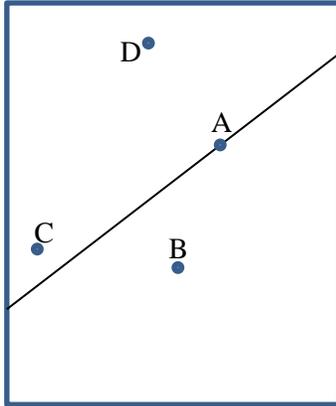
In geometry, a plane has no thickness but extends indefinitely in all directions. Planes are usually represented by a shape that looks like a parallelogram. Even though the diagram of a plane has edges, you must remember that the plane has no boundaries. A plane is named by a



single letter (plane p) or by three non-collinear points (plane ABC).

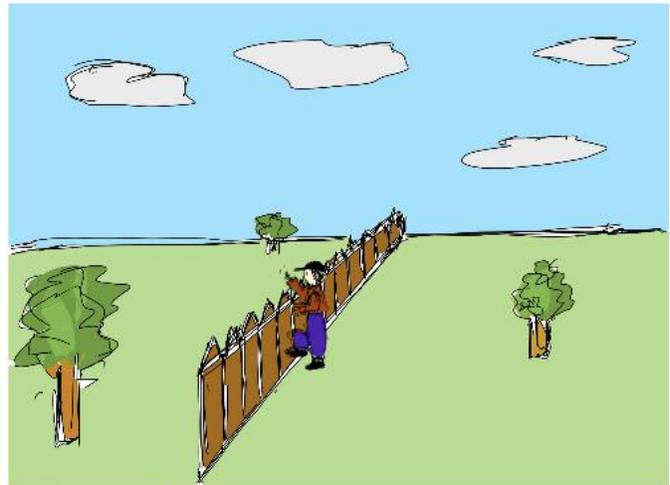


Take a sheet of paper and draw a straight line. Use ruler. Draw the line from one side of



paper to the other. This line divides a plane into two parts, two half-planes. Both, plane and line have no ends.

If a point is marked on this plane, the point can belong to the line or not. Points C and D belong to the same half-plane, they can be connected without crossing the line. Points C and B belong to two different half-planes, they can't be connected without crossing the line.

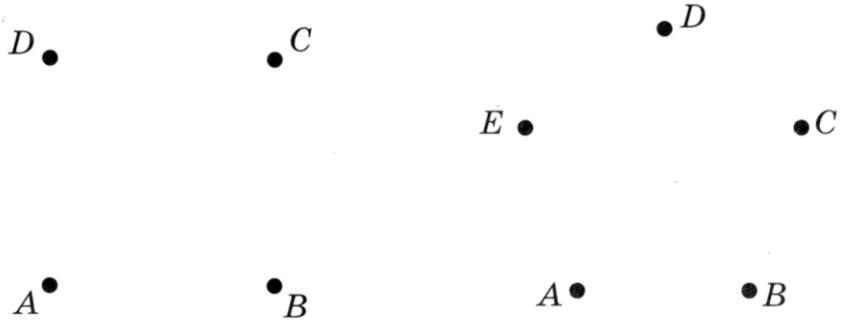


Problem 4. In to how many parts two lines can divide a plane?

3 lines can divide a plane? 4 lines? 5 lines?

Problem 5. How many points of intersection will three non-parallel line produce when no more than two lines intersect in one point? Same problem for 4 lines, 5 lines, n lines?

Problem 6. How many lines can be drawn through 4 points so that each line passes through 2 points? How many lines can be drawn through 5 points? n points?



Problem 7. The intersection of two straight lines is a point. What is the intersection of a plane and a line which doesn't belong to the plane? Two planes? (intersection is a common part of two sets of objects, in our case, points)

Exercises:

1. Into how many pieces you can cut a crepe with only three straight cuts?
2. Into how many pieces you can cut a watermelon with only three straight cuts?
3. Watermelon was cut into 4 pieces and eaten. There are five crusts left. How that can happen?
4. On a graph paper draw a square 4×4 . Divide it into four equal parts. How many different ways you can find?
5. Can you divide a triangle into four equal parts?
6. Four countries have a shape of triangles. Each of the country has borders with three other countries. Draw a possible map of all four.
7. Let's measure the thickness of a sheet of paper. We have an unpacked box of paper, a ruler with marks every 1 cm, and a scale. (Scale is not necessary, but can be very helpful). How can we do it?

