GAME NUMBERS 1: A GAMUT OF GAMES

JADE NINE

Combinatorial Games

Welcome to the first week of Game Numbers! Before we talk about numbers, we're going to talk about games, and I'll give you a few examples of games that we'll be playing, so you get a chance to work on figuring them out.

The games we'll be playing all have some important properties, some of which separate them from other types of games common in the study of game theory:

- Two-player: these games are all competitive two-player games
- Turn-based: players make a move inside the game board, after which it becomes the other player's turn
- Perfect information: everyone can see the board, and knows exactly what's going on (there are no hidden cards or secret decisions)
- Symmetric (also called *impartial*): both players have the same opportunities for play moves in the game - the only difference between the players is who goes first and who goes second. In a symmetric game, the only way to tell who is winning is to know whose turn it is, not who is which color, who's on what side, etc.

Such games are called combinatorial games. Here is a short diagram to illustrate a few types of two-player games, with examples (prisoner's dilemma and poker are not exactly turn-based, though poker depends on the version you're playing):

	Perfect Information	Imperfect Information
Symmetric	Combinatorial Games	Prisoner's Dilemma
Asymmetric	Chess, Checkers	Poker, Battleship

I will now introduce a few games. In class we will discuss ways that you can play these games, including a basic online method we'll be using to play each other during class.

Although there are variations, in combinatorial games it is standard to define the **object** of the game as **making the last move**. In other words, the winner of the game is the one who makes the last possibe move in the game so that the other player has no more possible moves to make. Combinatorial games are designed so that this sort of endpoint is the final goal of the game (you can consider it a form of standardization). Obviously, given this description, passing is not allowed in combinatorial games.

Most of the names of the following games are standard, but some are not. We'll talk about this in class.

Date: June 28, 2020.

1. Nim

The most famous and classic of combinatorial games, Nim goes like this:

- There is a pile of stones. On one's turn, one may take as many stones from the pile as one likes. (Negative stones are not allowed, you can only take as many as are available in the pile.)

Obviously, the winning move in this game is to take all the stones. Then your opponent is left with a blank pile, or a pile of 0, and cannot make any move, therefore they lose.

So what's the catch here?

The trick is that you can put multiple piles into the game, and each player must choose one pile to take stones from. Here, then, is a better definition of nim:

- There are piles of stones. On one's turn, one may take as many stones as one likes *but only from one pile.*

2. Cookies

There is a pile of cookies. On one's turn, one may eat one or two cookies from the pile.

3. Chocolate

There is a rectangular chocolate bar, an m-by-n grid of squares. On one's turn, one may split the bar along one of its lines to split it into two smaller bars.

When there is more than one chocolate bar available, one must choose one bar to split (you can't pick up two separate bars and split them both at the same time).

4. Kayles

There is a row of candles. On one's turn, one may blow out one of the candles, or two adjacent candles.

To give an example, suppose we start with a row of four candles. You take the first move, and you blow out the two candles in the center. Then there are two candles that remain lit, one on either end of the row. I can blow out one of these candles; I can't blow out both, because they are not adjacent candles (there's an empty space between them). I blow out the candle on the left. Then you blow out the candle on the right and you win, because all the candles are out now.

5. Grundy's Game

There is a tank full of fish. On one's turn, one may separate the fish into two different tanks that have a different number of fish in each tank.

If there is more than one tank of fish available, you must choose one tank to target, and then separate the fish of that tank into two smaller tanks. Again, the resulting tanks must have a different number of fish (though the number of fish in each tank may be the same as some other tank that you didn't touch).

For example, if we start with a tank of four fish, then you could split the fish into a tank of 1 and a tank of 3, in which case I target the tank of 3 and separate the fish into a tank of 1 and a tank of 2. Then I win, because you cannot target any tank (you can't separate a tank of 2, because the result would be two tanks of 1, which is forbidden by the game rules).

6. What's In The Combo

A burger joint has a stock of burgers, fries, and drinks (they all come in units of uniform size). On one's turn, one may take a bunch of burgers, a bunch of fries, a bunch of drinks, or a bunch of combos (where a combo is one burger, one fries, and one drink).

For example, suppose the joint has a stock of 2 burgers, 3 fries, and 3 drinks. On your turn you take two combos, leaving behind 1 fries and 1 drink. On my turn, I can't take any combos because there are no burgers, so I have to take fries or a drink. I take the drink. You then take the fries, and you win, because there's nothing left for me to take.

(This game is similar to one known as Wythoff's Game.)

7. Catch These Hands

On one's turn, one draws a pair of arrows, each of which can point up, up-right, right, down-right, down, down-left, left, or up-left (the two arrows you draw are not allowed to point in the same direction). You may not draw a pair of arrows whose directions are "almost-matching" to a pair already drawn. Two pairs of arrows are "almost-matching" if the arrows can be corresponded so that each one points in almost-the-same direction: for example, the pair (up, up-right) almost-matches the pair (up, right) because you can correspond up with up and up-right with right, where up matches with up and up-right points in almost-the-same direction as right (they are effectively adjacent directions). (up,up-right) and (up,up-left) also almost-match, because you can correspond up with up-left and up-right with up. But (up,down-right) and (up,down-left) do not macth, because you would have to correspond up with up and then down-right and down-left are too far apart to be almost-matching.

We can discuss this one more in class if the rules seem confusing, but the idea is that eventually one can no longer possibly draw an arrow pair that does not almost-match one already drawn.

"Hands", by the way, because you can think of them as hands on a clock, sort of. The difference is that there are 8 directions here, not 12.

8. Chomp!

On one's turn, one writes down a two-digit number (in this game, the digit 0 is not allowed). You cannot write down a number whose digits are both greater than or equal to the corresponding digits a number already written down.

For example, let's say I go first, and I write down 52. Then you can't write 63, because in the tens digit $6 > 5$ and in the ones digit $3 > 2$. You also can't write down 55, 74, 82, etc. You decide to write down 23. Then I write down 12. You then write down 11, and you win, because there is no two-digit number that uses digits less than 1 (remember that I can't write down 10 because 0 is not allowed as a digit in this game).

Of course, if the first player just writes down 11, then they win the game.

What's the catch?

The trick is to write down colored numbers, starting with a fixed collection of colors, for example red and purple. Then, on one's turn, one writes down a two-digit number, and the number you write down cannot clash with a number of the same color (where "clash" is based on the definition of the game above). In this case, if you write down red 11, then I write down purple 11 and I win, because you can't write down any red or purple number that doesn't clash with the 11s. So now what is the strategy?

9. Princess and the Roses

There is a garden of roses of several colors. On one's turn, one may take roses from the garden to give to the Princess, who is in charge of approving grants for a public arts project you are trying to secure. The Princess is only interested in endorsing the project of the person who brings the last rose from the garden. The rule of this game is that you are only allowed to take roses of different colors - you are not permitted to bring the Princess more than one rose of the same color on your turn. Essentially you are only allowed to bring the Princess a single, strictly multicolor "bouquet".

For example, suppose the garden has two orange roses, one red rose, and one white rose. You go first, and bring the princess the red and the white rose. Now I only have one choice, I can only bring one orange rose to the Princess. You then bring the Princess the second orange rose and you win, because there are no more roses left in the garden.

10. Arrows

A bunch of arrows, each of which are pointing up or down, are placed in a row from left to right. On one's turn, one may select an up-pointing arrow, and flip it together with all arrows to its right. When there are no more arrows pointing up, the game is over.

Homework

The homework for this week is just to familiarize yourself with each of the games above and try playing them. I will try to arrange times during the week where you can play with each other; I will also give advice during class on ways that you can play the games by yourself to test out how they work. See what strategies you can come up with!