Summer 2020, Math 4$\frac{1}{2}$

Lesson 6.

1. **Exponent.**

Exponentiation is a mathematical operation, written as $\boldsymbol{b^n}$, involving two numbers, the **base** *b* and the **exponent** *n*. When *n* is a positive integer, exponentiation corresponds to repeated multiplication of the base: that is, $\boldsymbol{b^n}$ is the product of multiplying *n* bases:

$$b^n = \underbrace{b \times \cdots \times b}_{n}$$

In that case, $\boldsymbol{b^n}$ is called the *n*-th power of *b*, or *b* raised to the power *n*.

The exponent indicates how many copies of the base are multiplied together. For example, $3^5 = 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 = 243$. The base 3 appears 5 times in the repeated multiplication, because the exponent is 5. Here, 3 is the *base*, 5 is the *exponent*, and 243 is the *power* or, more specifically, *the fifth power of 3*, *3 raised to the fifth power*, or *3 to the power of 5*.

Properties of natural exponent:

If the same base raised to the different power and then multiplied:

$$b^3 \cdot b^4 = (b \cdot b \cdot b) \cdot (b \cdot b \cdot b \cdot b \cdot b) = b \cdot b \cdot b \cdot b \cdot b \cdot b \cdot b \cdot b = b^7$$

Or in a more general way:

$$b^n \cdot b^m = b^{n+m}$$

If the base raised to the power of n then raised again to the power of m:

$$(b^2)^3 = (b \cdot b)^3 = (b \cdot b) \cdot (b \cdot b) \cdot (b \cdot b) = b^{2 \cdot 3}$$

$$(b^m)^n = b^{mn}$$

If we want to multiply $b^n = \underbrace{b \cdot b \cdot b \ldots \cdot b}_{n\ times}$ by another b we will get the following expression:

$$b^n \cdot b = \underbrace{b \cdot b \cdot b \ldots \cdot b}_{n\ times} \cdot b = \underbrace{b \cdot b \cdot b \cdot b \ldots \cdot b}_{n+1\ times} = b^{n+1} = b^n \cdot b^1$$

In order to have the set of power properties consistent, $b^1 = b$ for any number b.

If we multiply b^n by 1, we won't change anything, so we can write

$$b^n \cdot 1 = b^{n+0} = b^n \cdot b^0$$

In order to have the set of power properties consistent, $b^0 = 1$ for any number $b \neq 0$

If two different bases raised to the same power, then:

$$(a \cdot b)^3 = (a \cdot b) \cdot (a \cdot b) \cdot (a \cdot b) = a \cdot a \cdot a \cdot b \cdot b \cdot b = a^3 b^3$$

$$(a \cdot b)^n = a^n b^n$$

1.

$2^3 \cdot 2^2 =$	$(2^3)^2 =$
$5^2 \cdot 5 =$	$(3^7)^2 =$
$2^5 \cdot 2^3 \cdot 2 =$	$(n^5)^3 =$

2. Write the following expressions as a product or power:

$$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2;$$

$$2 + 2 + 2 + 2 + 2;$$

$$a \cdot a \cdot a;$$

$$a + a + a;$$

$$\underbrace{x \cdot x \cdot \ldots \cdot x}_{20\ times};$$

$$\underbrace{x + x + \cdots + x}_{20\ times}$$

3. Write the following expressions in a shorter way:

   Example: $7 \cdot 7 \cdot 7 \cdot 8 \cdot 8 \cdot 8 \cdot 8 \cdot 9 \cdot 9 \cdot 9 \cdot 9 \cdot 9 = 7^3 \cdot 8^4 \cdot 9^5$

   $2 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 7 \cdot 7;$

   $\underbrace{3 \cdot 3 \cdot \ldots \cdot 3}_{n\ times} \cdot \underbrace{5 \cdot 5 \cdot \ldots \cdot 5}_{m\ times}$

   $\underbrace{(-4) \cdot (-4) \cdot \ldots \cdot (-4)}_{k\ times} \cdot \underbrace{6 \cdot 6 \cdot \ldots \cdot 6}_{l\ times}$

4. Compare the numbers:

   $a.\ 5^3 \quad 5 \cdot 3$ $\qquad$ $b.\ 12^2 \quad 12 \cdot 2$

   $c.\ 2^5 \quad 5^2$ $\qquad$ $d.\ 3^4 \quad 4^3$

   $e.\ 5^3 \quad 5 \cdot 3$ $\qquad$ $f.\ 2^4 \quad 4^2$

5. Compare:

   $a.\ -3^2 \ldots (-3)^2;$ $\quad$ $b.\ 3^2 \ldots (-3)^2$ $\quad$ $c.\ -3^3 \ldots (-3)^3$ $\quad$ $d.\ 3^3 \ldots (-3)^3$

6. Positive or negative number will be
   a. Even power of a positive number.
   b. Even power of a negative number.
   c. Odd power of a positive number.
   d. Odd power of a negative number.

7. Simplify:

   $3 \cdot 3^4(-3)^2;$ $\qquad$ $2 \cdot 3^2 \cdot 5^3 \cdot (-4 \cdot 3 \cdot 5^2);$

   $2^5 \cdot 2(-2^2)c^{4-1}c^3;$ $\qquad$ $0.5a(-b)^6 \cdot 10a^2b^2;$

   $5^3 \cdot 5(-5^5)5^3 \cdot 5;$ $\qquad$ $\frac{1}{6}(-5)^3 5 \cdot 3 \cdot (-6 \cdot 5 \cdot 3^3);$

8. Compare:

Example:

$$28^2 \quad 1000; \qquad 28^2 = 28 \cdot 28 < 30 \cdot 30 = 900; 900 < 1000,$$

therefore $28^2 < \ 1000$

$a.\ 28^2 \quad 1000;$ $\qquad b.\ \ 48^2 \quad 3000;$
$a.\ 42^2 \quad 1500;$ $\qquad b.\ \ 67^2 \quad 3500;$

9. Write the number which extended form is written below;

$a.\ 2 \cdot 10^3 + 4 \cdot 10^2 + 5 \cdot 10 + 8;$ $\qquad b.\ 7 \cdot 10^3 + 2 \cdot 10^2 + 0 \cdot 10 + 1;$
$c.\ 9 \cdot 10^3 + 3 \cdot 10 + 3;$ $\qquad e.\ 4 \cdot 10^3 + 1 \cdot 10^2 + 1 \cdot 10 + 4;$

10. Evaluate and compare:

$a.\ \ 2 \cdot 10^3 \ \text{ and } \ (2 \cdot 10)^3;$ $\qquad b.\ \ 3 \cdot 2^2 \ \text{ and } \ (3 \cdot 2)^3;$
$c.\ \ 2 \cdot 5^3 \ \text{ and } \ (2 \cdot 5)^3;$ $\qquad b.\ \ 12: 2^2 \ \text{ and } \ (12: 2)^2;$

11. Put digits instead of stars to create the true equalities. How many answers does each problem have?

$a.\ (2\,*)^2 =**\,1;$ $\qquad b.\ (7\,*)^2 =***\,5;$ $\qquad c.\ (3\,*)^2 =***\,6;$
$d.\ (2\,*)^2 =**\,9;$ $\qquad e.\ (**)^2 =**\,4;$

12. Simplify the following expressions:

$a.\ \ 2^4 + 2^4;$ $\qquad d.\ \ 3^2 + 3^2 + 3^2;$

$b.\ \ 2^m + 2^m;$ $\qquad e.\ \ 3^k + 3^k + 3^k;$

$c.\ \ 2^m \cdot 2^m;$ $\qquad f.\ \ 3^k \cdot 3^k \cdot 3^k;$

Haw many choices do we have?

There are 5 chairs and 5 kids in the room. In how many ways can kids sit on these chairs? The first kid can choose any chair. The second kid can choose any of the 4 remaining chairs, the third child has a choice between the three chairs, and so on. Therefore, there are $5 \times 4 \times 3 \times 2 \times 1$ ways how all of them can choose their places. Thus obtained long expression, $5 \times 4 \times 3 \times 2 \times 1$, can be written as 5!. By definition:

$$5 \times 4 \times 3 \times 2 \times 1 = 5! \qquad or \quad n \times (n-1) \times (n-2) \times \ldots \times 3 \times 2 \times 1 = n!$$

Write the following expressions as a factorial and vice versa:

Example: $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 7!$, $4! = 4 \times 3 \times 2 \times 1$

$10 \times 9 \times 8 \times \ldots \times 3 \times 2 \times 1 =$

$6! =$

$b \times (b-1) \times (b-2) \times \ldots \times 3 \times 2 \times 1 =$

$c! =$

13. Simplify the following fractions:

$\frac{5!}{7!} =$

$$\frac{n!}{(n-2)!} =$$

14. How many different ways are there to put 64 books on the shelf?

15. There are 20 students in the 5th grade. They have to choose a president, and a vice president of the class. How many different ways are there to do it?

16. There are 20 students in the 5th grade. They have to choose a team of two students to go to the math competition. How many different ways are there to do it?

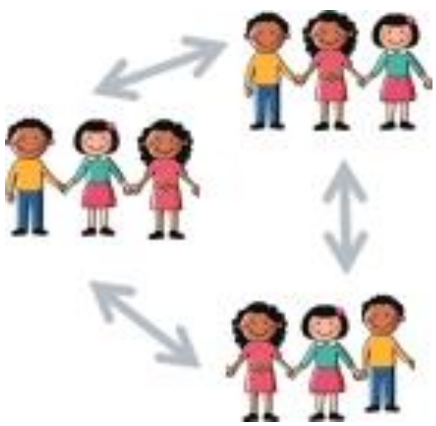
How many different 3-digit numbers can we create using 8 digits, 1, 2, 3, 4, 5, 6, 7, and 8 without repetition of the digits, i.e. such numbers that only contain different digits?

How many different ways are there to choose a team of 3 students out of 8 to participate in the math Olympiad.

What are the similarities in these two problems?

Can you see the difference between them?

In both cases, we have 8 possible ways to choose the first item (digit or student), 7 possible ways to choose the second item, and 6 different ways to choose the third one. So,



there are $8 \cdot 7 \cdot 6$ different 3-digit numbers created from digits 1, 2, 3, 4, 5, 6, 7, and 8 and $8 \cdot 7 \cdot 6$ different teams of 3 students out of 8. Or not?

We can create numbers 123, 132, 213, 231, 321, 312 and they are all different numbers. If we chose Mike, Maria, and Jessika, a team of 3 students for the math Olympiad, it doesn't matter in which order we wrote their names.

In the first case, we have $8 \cdot 7 \cdot 6$ ways to create a 3-digit number out of 8 digits. In the second case for each group of 3 kids we will count 6 times (3! – number of ways to put 3 kids in line) more possible choices than there really are. So the total number of the way to choose the teem is

$$\frac{8 \cdot 7 \cdot 6}{3 \cdot 2 \cdot 1}$$

17. In the restaurant, there are 3 choices of starters, 4 choices of entrees and 5 choices of tasty desserts in the fix price dinner menu. How many different ways are there to fix a dinner for the restaurant's clients?
18. How many two-digit numbers can be composed from digits 1, 2, 3 without repetition of digits?
19. How many two-digit numbers can be composed from digits 1, 2, 3, if repetition is allowed?

20. Peter took 5 exams at the end of the year. Grade for exams are A, B, C, D. How many different ways are there to fill his report card?

21. I have 5 new books to read during my 5 days' vacation. I want to read 1 book every day. How many different ways are there for me to read these 5 books? How many ways would be there if I would have only a 3 days' long weekend to read them? In this case I will be able to read only 3 books in total.