Summer 2020, Math $4\frac{1}{2}$

Lesson 5.

Equalities: equations and identities.

terms

$$\underbrace{3x\text{-}2ab\text{-}10+4x+24\text{-}5ab+75y}_{\text{expression}}$$

Expressions.

Mathematical expressions are the mathematical phrases that contain numbers, symbols, letters. Terms can be numbers or numbers combined with letters. In the latter case letters are called "variables" and a number is called "coefficient". If the term contains only the number than it's called "constant". In the term $2ab$ number 2 is a coefficient and a, and b are variables. The "like terms" in the expression above are ones that have the same variable. All constants are like terms as well. To simplify the expression all like terms should be combined. In other words, all constant should be added together as well as all terms which contain the same variables. For the expression above

$$3x-2ab-10+4x+24-5ab+75y=3x+4x-2ab-5ab+75y-10+24=$$
$$=7x-7ab+75y+14$$

Is there any difference between two following equalities?

$a(b+c)=ab+ac$
$a+2=6$

Letters a, b, and c in both these expressions are called *variables*, we can put any number (whole or fraction) into it. In the first case the equality is still a true expression for any a, b, and c, this is a distributive property of addition.
The second expression is a true expression for only one value of $a=4$ and we call this kind of expressions "an equation". An equation is the problem of finding values of some variables, called *unknowns*, for which the specified equality is true. We have to solve the equation to find the value of an unknown variable.

How to solve an equation?

An equation is a statement that the values of two mathematical expressions are equal (indicated by the sign =). For example, in the equation

$$3x - 5 = 4x - 7$$

one expression $(3x - 5)$ equals to the expression $(4x - 7)$. Solving the equation, means to find such number x that will make the equality true.

In order to do it first we have to combine all like terms of the expressions. Because both side of the equation are equal than the equal terms can be added (or subtract) to (from) both sides and it will not change the equality rule:

$$3x - 5 = 4x - 7$$
$$3x - 3x - 5 = 4x - 3x - 7$$
$$-5 = x - 7$$
$$-5 + 7 = x - 7 + 7$$
$$2 = x$$

It is not really necessary to write all these sequential statements, we just need to rewrite the term on another side of the equation with the opposite sign (but you have to know why this is the right way to do). Both sides of the equation can be divided (or multiplied) by the same number (or term) and as the result we will get the equality again.

$$4 \cdot (x + 5) = 12$$

$$\frac{4 \cdot (x + 5)}{4} = \frac{12}{4}$$

$$x + 5 = 3$$

$$x + 5 - 5 = 3 - 5$$

$$x = -2$$

1. Simplify the following expressions:

   $a.\quad 2 + 3a + xy + 4 - a + xy - 6 =$

   $b.\quad d - 4 + t + t + 32 + 3d =$

   $c.\quad x + 5s - 3s + 2x =$

2. Solve the following equations:

   $a)\quad b - \frac{1}{6} = \frac{1}{6}, \qquad b)\quad \frac{1}{6} + x = \frac{1}{2}, \qquad c)\quad c \cdot 4 = \frac{1}{5}, \qquad d)\quad a - \frac{4}{9} = \frac{1}{3}$

3. The fourth grade is going on a school trip. Every student had to bring in \$64 for the trip to cover all costs. Unfortunately, 3 students could not participate on the trip. Therefore,

every student who went on the trip had to bring in $4 more so those who did not go could get their money back. How many students went on the trip?

4. On the lawn grew 35 yellow and white dandelions. After eight whites flew away, and two yellows turned white, there were twice as many yellow dandelions as white ones. How many whites and how many yellow dandelions grew on the lawn at the beginning?

5. 25 identical thick books or 45 identical thin books can fit on a bookshelf. Will there be enough space on a bookshelf for 20 thick and 9 thin books?

6. A swimming pool can be filed by one pipe in 10 hours or by another pipe in 15 hours. How long it will take to fill up the pool with both pipes opened?

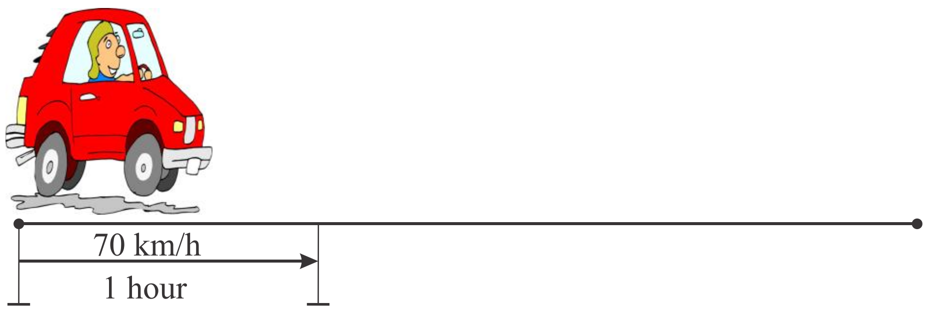
7. A swimming pool can be filed with one pipe in 10 hours. Full pool can be drain out with another pipe in 20 hours. How long it will take to fill up the pool with opened drain pipe?

8. A farmer has a cow, a goat and a goose. The cow and the goat will eat all the grass on his meadow in 45 days, the cow and the goose will eat all the grass on the same meadow in 60 days, and the goat and the goose will eat all the grass on the meadow in 90 days. How many days will it take them altogether to eat all the grass on the meadow? (we assume that the new grass is not growing.)

Speed, time, and distance.

Car was moving for 3 hours with the speed of 70 km/h. How far did it travel? In this kind of problems in math we always assume that the car (or any other moving object) is moving with the constant speed along the straight line. Of course, this is seldom case in the actual reality, and in physics you will be studying the laws of motion in a more profound way.

Let’s denote the speed of the car v, the time during which the car was moving t, and the distance it travelled, S. These letters are usually used for speed, time and distance, but you can use any other letters as well.



$$S = v \times t = vt$$

If $v = 70km/h$ and $t = 3h$, then $S = \frac{70k}{h} \times 3h = 70km$.

This is simple. If we know two out of three parameters, we always can find the third one.

$$S = vt;$$

$$v = \frac{S}{t};$$

$$t = \frac{S}{v};$$

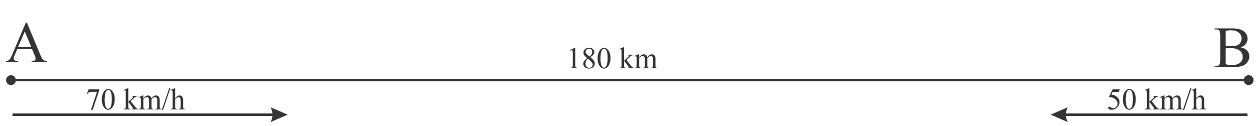
9. Peter was walking for 15 minutes with the speed of 5km/h. How far did he go?

10. The speed of the boat in a still water on a lake is 12 km/h. The speed of the river flow is 3 km/h. How many hours does the boat need to go from the city A to the city B if the distance between the two cities is 45 km and the city A is up on the river, i.e. the river flows from A to B?

How many hours does this boat need to go back from the city B to the city A?

11. The speed of the boat going downstream the river is 19 km/h, and the speed of the same boat going upstream this river is 15 km/h. What is the speed of the river stream and what is the speed of the boat in a still water on a lake?

12. Two cars start moving towards each other at the same time from the two cities, A and B. The distance between the cities is 180 km. The speed of the car that departed from the



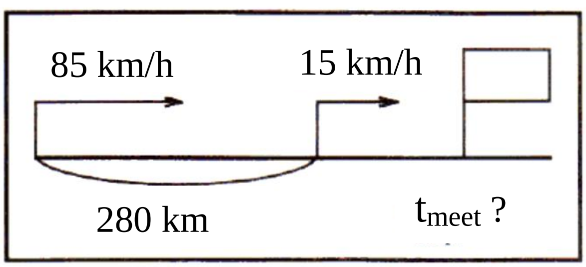
city A is 50 km/h, the speed of the car that left from the city B is 70 km/h. In how many hours will they meet? How far from the city A they will meet?

13. Two cars start moving at the same time in the same direction from cities A and B, as shown in the picture below.

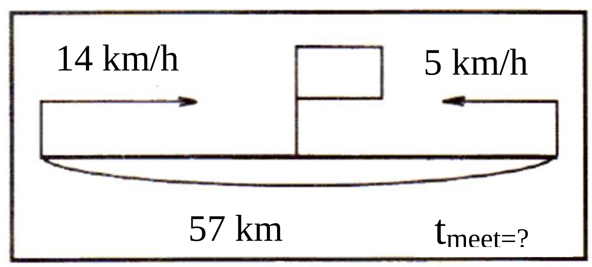
How many hours will it take for the faster car to catch up with the slower car? How far from the city A will they meet?

14. For the four pictures below, come up with the problem and solve it.

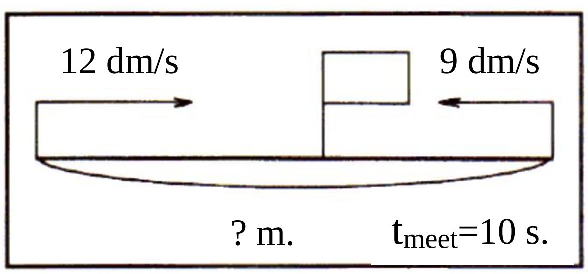
a)



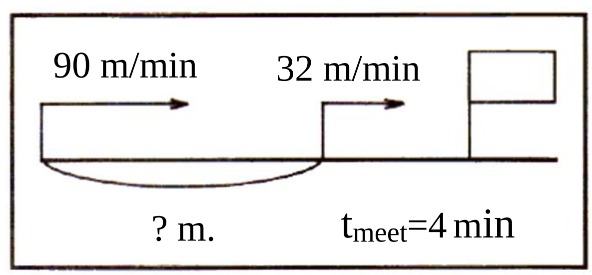
c)



b)



d)



15. Every morning Peter walks his dog. He goes with the speed of 4 km/h and his walk is usually 8 km long. His dog is running around with the speed 7 km/h. What dastans does dog run?

16. Two bicyclists start 100 miles apart, and head towards each other, each one going 10 mph. At the same instant, a fly leaves the first bike and flies at 20 mph to the second.

When it gets there, it immediately turns around and heads back to the first. Then it repeats, going back and forth between the two bikers. By the time they reach each other, how far will the fly have travelled?

17. Andrew is walking along a narrow bridge. When Andrew passes exactly 1/3 of the length of the bridge, he notices a cyclist on the road to the bridge heading after him. If Andrew will starts walking toward the cyclist, they will meet at the beginning of the bridge. If he will continue toward the end of the bridge, the cyclist will catch up with him at the end of the bridge. How many times is the speed of the cyclist higher than the speed of the walker?