Lesson 2

Distributive property.

$$a \cdot (b + c) = a \cdot b + a \cdot c$$

Example:

Farmer put green and red grapes into boxes. Each box contains 5lb of grapes. How many pounds of green and red grapes altogether did farmer put into boxes if he had 10 boxes of green and 8 boxes of red grapes? Is there any difference between these 2 expressions:

$$5 \cdot (10 + 8) \quad or \quad 5 \cdot 10 + 5 \cdot 8$$

Using the definition of multiplication and commutative property of addition:

$$5 \cdot (10 + 8) = (10 + 8) + (10 + 8) + (10 + 8) + (10 + 8) + (10 + 8)$$
$$= 10 + 10 + 10 + 10 + 10 + 8 + 8 + 8 + 8 + 8 = 5 \cdot 10 + 5 \cdot 8$$

$$a \cdot (b + c) = \underbrace{(b + c) + (b + c) + \cdots + (b + c)}_{a\ times} = \underbrace{b + b + .. + b}_{a\ times} + \underbrace{c + c + \cdots + c}_{a\ times} = a \cdot b + a \cdot c$$

Factorization.

In mathematics factorization is a decomposition of on object into a product of other objects, or representation of an object as a product of 2 or more objects, which called 'factors'. For example, we can represent the expression $a \cdot b + a \cdot c$ as a product of a and expression $(b + c)$.

$$a \cdot b + a \cdot c = a \cdot (b + c)$$

Or in a numerical expression:

$$4 \cdot 5 + 4 \cdot 3 = 5 + 5 + 5 + 5 + 3 + 3 + 3 + 3 = (5 + 3) + (5 + 3) + (5 + 3) + (5 + 3)$$
$$= 4 \cdot (5 + 3)$$

Or a number can be representing as product of two or more other numbers, for example:

$$40 = 4 \cdot 10, \quad 36 = 6 \cdot 6$$

Does any natural number can be represented as a product of 2 or more numbers besides 1 and itself? Natural numbers greater than 1 that has no positive divisors other than 1 and itself are called prime numbers.

Prime factorization or integer **factorization** of a number is the determination of the set of **prime** numbers which multiply together to give the original integer. It is also known as **prime** decomposition.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Prime factorization process:

Prime factors of 168 are $2, 2, 2, 3, 7$ and prime factors of 180 are $2, 2, 3, 3, 5$,

$$\begin{array}{r|l} 168 & 2 \\ 84 & 2 \\ 42 & 2 \\ 21 & 3 \\ 7 & 7 \\ 1 & \end{array} \qquad \begin{array}{r|l} 180 & 2 \\ 90 & 2 \\ 45 & 3 \\ 15 & 3 \\ 5 & 5 \\ 1 & \end{array}$$

$$2 \cdot 2 \cdot 2 \cdot 3 \cdot 7 = 168; \quad 2 \cdot 2 \cdot 3 \cdot 3 \cdot 5 = 180$$

Any natural number has single unique prime factorization.

For Halloween the Jonson family bought 168 mini chocolate bars and 180 gummi worms. What is the largest number of kids between whom the Jonson can divide both kinds of candy evenly?

To solve this problem we have to find a number which can serve as a divisor for 168 as well as for 165. There are several such numbers. The first one is 2. Both piles of candy can be evenly divided between just 2 kids. 3 is also a divisor. The Jonson family wants to treat as many kids as possible with equal numbers of candy. To do this they have to find the Greatest Common Divisor (GCD), the largest number that can be a divisor for both (168 and 180) amounts of candy. Let's take a look at all prime factors of 168 and 180.

$$168 = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 7; \qquad 180 = 2 \cdot 2 \cdot 3 \cdot 3 \cdot 5$$

$$\begin{aligned} 168 &= 2 \cdot 2 \cdot 2 \cdot 3 \qquad \cdot 7 \\ 180 &= 2 \cdot 2 \qquad \cdot 3 \cdot 3 \cdot 5 \end{aligned}$$

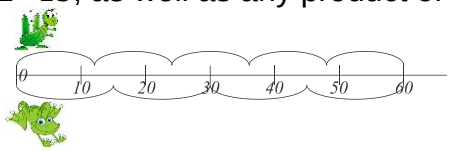
Prime factors of 168 are 2, 2, 2, 3, and 7. Any of these numbers as well as any of their products can be a divisor for 168. The same goes for the set of prime factors of 180, which are 2, 2, 3, 3, and 5. It is easy to see that these two sets have common elements. It means that both numbers are divisible by any of these common elements and any their products. The largest product is the product of all common elements. This largest product is GCD (GCF).

$$168 \div 12 = 14$$

$$180 \div 12 = 15$$

A grasshopper jumps a distance of 12 centimeters each jump. A little frog jumps a distance of 15 centimeters each jump. They start hopping at the same time from the same point 0 and jump along the big ruler. What is the closest point on the ruler at which they can meet?

There are places on the ruler that both of them can reach after some number of jumps. One of such places is, of course, $12 \cdot 15$ cm. A grasshopper can make 15 jumps while a little frog can make only 12 jumps. Will $12 \cdot 15$ be the only place where they can meet or there are some other places? If this is the case, we have to find a number that is divisible by both 12 and 15. Take into account that $12 \cdot 15$, as well as any product of $12 \cdot 15$ is divisible by both 12 and 15.



Is there are any number which is less then $12 \cdot 15$ and still divisible by 12 and 15? Prime factorization of 12 and 15:

$$\begin{aligned} 12 &= 2 \cdot 2 \cdot 3 \\ 15 &= \qquad 3 \cdot 5 \end{aligned}$$

The product of $2 \cdot 2 \cdot 3 \cdot 5$ will be divisible by both 12 and 15:

$$2 \cdot 2 \cdot 3 \cdot 5 = 60$$

Exercises:

1. Compute using the distributive property and using the order of operation:

   $2 \cdot (31 + 22) =$
   $3 \cdot (35 - 20) =$
   $15 \cdot (6 + 4) =$

2. Compute by the most convenient way, use the distributive or/and other properties of addition/multiplication:

   $23 \cdot 15 + 15 \cdot 77 =$

   $79 \cdot 21 - 69 \cdot 21 =$

   $340 \cdot 7 + 16 \cdot 70 =$

   $250 \cdot 61 - 25 \cdot 390 =$

   $67 \cdot 58 + 33 \cdot 58 =$

   $55 \cdot 682 - 45 \cdot 682 =$

3. Find the prime factorization of the number:

$$2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot 8 \cdot 9 \cdot 10$$

4. Find the prime factorization of:

   $111,1111.$

5. The product of two naturel numbers m and n is 1000. Both numbers are not divisible by 10. Find these numbers.

6. Dunno boasted ability to multiply in the mind. To test it, Doono suggested writing some number, multiplying its digits, and saying the result. “1210” Dunno
   immediately blurted out, only having had time to write down the number. "It cannot be," - replied, thinking, Doono. How did he detect the error without knowing the source number? (Dunno and Doono are main characters in the book *The Adventures of Dunno and his Friends,* by Nikolay Nosov)

7. A florist has 36 roses, 90 lilies, and 60 daisies. What is largest amount of bouquets he can create from these flowers evenly dividing each kind of flowers between them?

8. On a track for remote-controlled racing cars, racing car A completes the track in 28 seconds, while racing car B completes it in 24 seconds. If they both start at the same time, after how many seconds will they be side by side again?

9. Mary has a rectangular backyard with sides of 48 and 40 yards. She wants to create square flower beds, all of equal size, and plant different kind of flowers in each flower bed. What is the largest possible size of her square flower bed?

10. Mary and Julia are twins. They invited 28 friends to their birthday party. Mary wrote 3 time as many invitation cards as Julia did. How many cards did Julia write?

11. A cow weighs 20 times as much as sheep weights. Together the cow and the sheep weight 2100 lb. How many pounds does the sheep weight? How many pounds does the cow weight?

12. Solve the puzzles (each letter represents a digit):

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  ELF        EAT        SEND       CIRCLE
+ ELF      + THAT     + MORE      CIRCLE
 FOOL       APPLE      MONEY     +CIRCLE
                                  SPHERE
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