Velocity and Acceleration

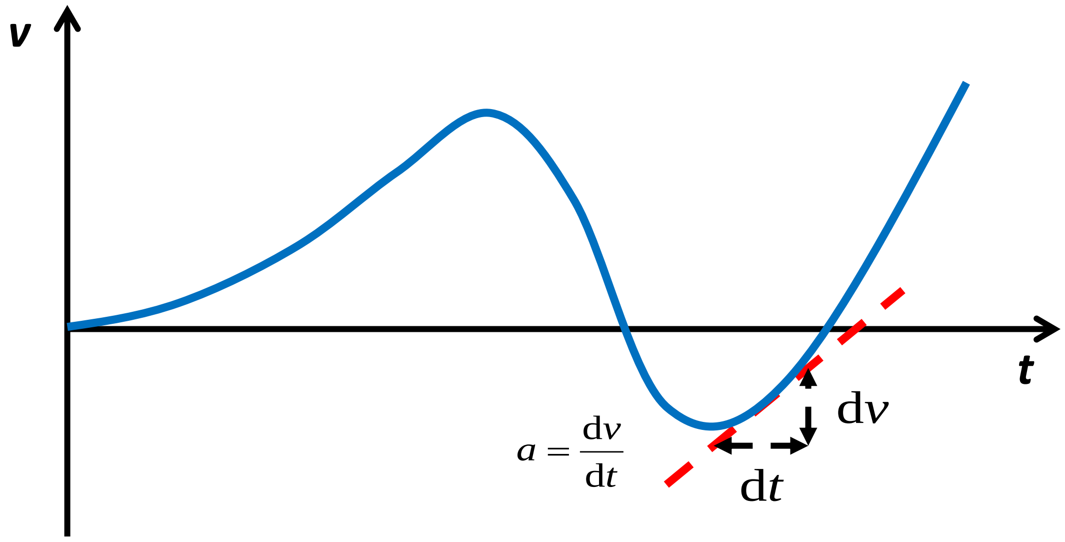
Last time, we defined **velocity** as a time derivative of a **position**:

$$v \underset{\Delta t \to 0}{=} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

Similarly, **acceleration** is the time derivative of **velocity**.

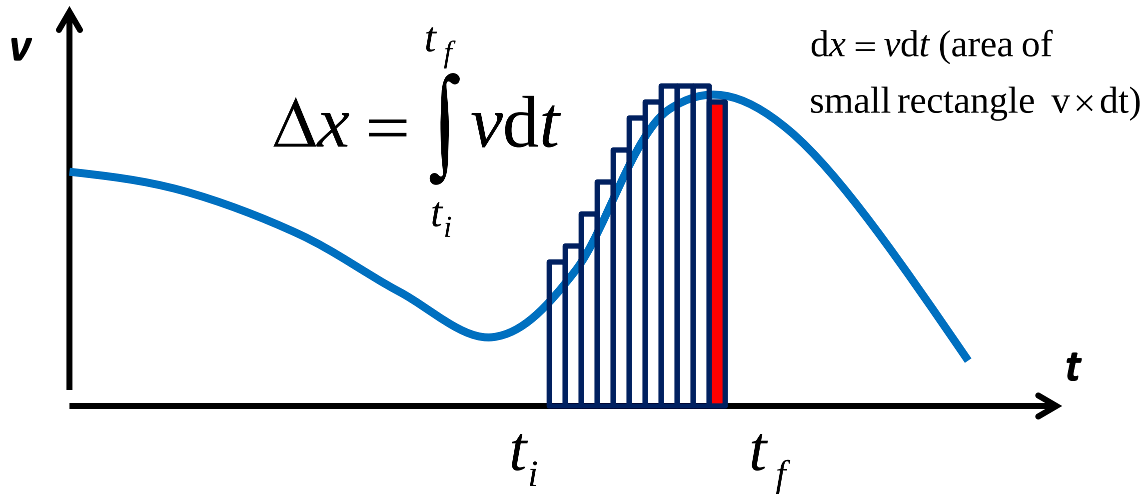
$$a = \frac{dv}{dt}$$

In other words, it is the rate of change of velocity, or local slope of the plot v(t) :



Position=Integral of Velocity
Velocity=Integral of Acceleration

If we know velocity at any moment of time, we can find how much the position changed by taking sum of its changes in little time intervals, dt. This sum is called ***time integral.***
It is equal to the area under the plot v(t), between initial and finite time:



SUMMARY:

$$v = \frac{\mathrm{d}x}{\mathrm{d}t}$$
derivative(slope)

$$a = \frac{\mathrm{d}v}{\mathrm{d}t}$$
derivative(slope)

x <u>position</u> **v** <u>velocity</u> **a** <u>acceleration</u>

$$\Delta x = \int_{t_i}^{t_f} v\mathrm{d}t$$
integral (area under plot)

$$\Delta v = \int_{t_i}^{t_f} a\mathrm{d}t$$
integral (area under plot)

Equations of Motion

- **Equation of Motion** gives position of a particle as a function of time.
- Motion with constant velocity is called ***uniform.*** **Equations of Uniform Motion in 1D**:

$$a(t) = 0$$

$$v(t) = v_0$$

$$x(t) = x_0 + v_0 t$$

Here $x_0 = x(0)$ and $v_0 = \mathrm{v}(0)$ are coordinate x and velocity v at time $t = 0$.

- Equations of **Constant-Acceleration Motion in 1D**:

$$a(t) = a$$

$$v(t) = v_0 + at$$

$$x(t) = x_0 + v_0 t + \frac{at^2}{2}$$

HOMEWORK

The table below shows velocity of a car ***v***, at various moments of time ***t*** after the start. Using these data:

a) Sketch the plot v(t);
b) Fill in the missing information (acceleration ***a*** and position ***x***), and plot them versus time. Remember that acceleration is the derivative of velocity (slope), and position is an integral of velocity (area under the plot).

t (s)	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
a (m/s^2)									
v (m/s)	0	10	18	25	31	36	39	40	40
x (m)	0								