Homework 12

Electrical capacitance and capacitors.

Last class we discussed electric capacitance. Let us consider the electrical circuit shown below:

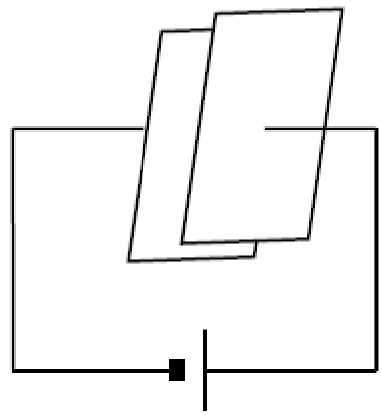


Figure 1.

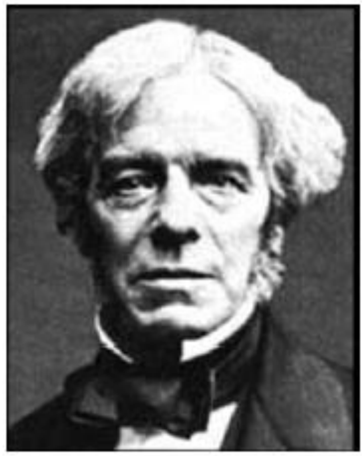
The circuit consists of two metal plates separated by a distance d and connected to a battery. The area of each plate is S. Since the plates are separated, it is natural to assume that no current will flow through the wire. This assumption is correct in a long time perspective, but for a short time, immediately after we have connected the battery, the current will flow. It happens, because in the first moment, electrons “do not know” that there is a disruption in the circuit and flow from the negative terminal of the battery to the plate which is connected to this terminal, while electrons from the other plate are being “sucked” to the positive terminal of the battery. The plates are getting electrically charged, and it takes some time. The charging current will flow until electrical potentials of the “negative” and “positive” plates are equal to the potentials of the negative and positive terminals of the battery. In other words, the charging current will flow until the potential difference (voltage) between the plates is equal to the battery voltage. The magnitude of the final charge on each plate Q is proportional to the voltage on the plates U:

$$Q = C \cdot U \qquad (1)$$

The coefficient C is called electrical capacitance. It depends on the area of the geometry of the plates. In our case –two parallel metal plates of same shape and size -the capacitance depends on the area of the plate ***S***, the distance ***d*** between the plates and the material between the plates (2),

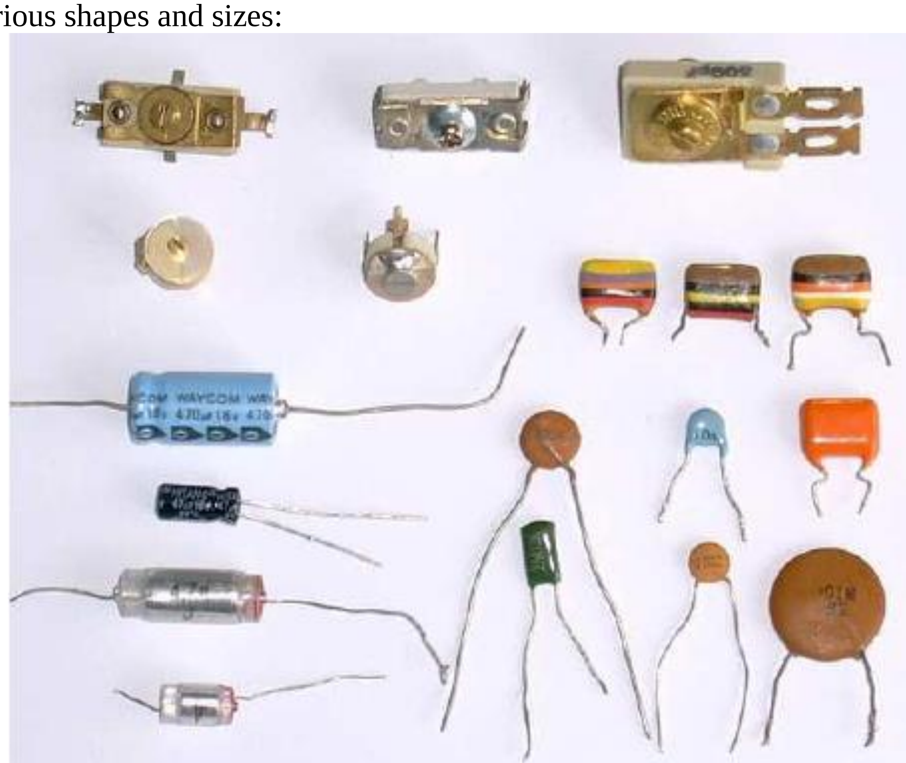
$$C = \frac{\varepsilon_0 \varepsilon S}{d} \qquad (2)$$

Where**,** ε_0=8.85 x10^{12}C/Vm is the parameter called “permittivity of vacuum”, ε is the dimensionless number which characterizes the material between the plates. This parameter is called dielectric constant of the material. $\varepsilon=1$ for vacuum. The unit of capacitance is Coulomb per Volt (1C/V) or Farad. This unit is named after famous English physicist and chemist Michael Faraday



Michael Faraday(1791-1867).

The electrical element which consists of separated metal plates is called capacitor. The capacitors can be of various shapes and sizes:



We learned that for parallel connection the resulting capacitance is the sum of the capacitances:

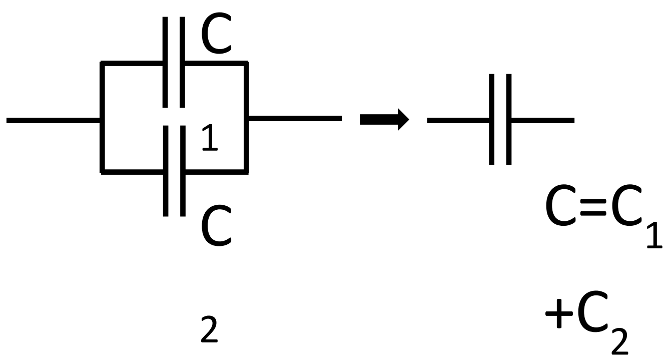


Figure 1. Parallel connection of the capacitors

Since the total voltage U on two elements connected in parallel is the same ($U_{total}=U_1=U_2$), but the total area of the capacitor plates is now the sum of the plate areas of the capacitors, we can expect that total charge on the "negative" (or "positive") plates is also the sum of the charges on the corresponding plates of two capacitors: $Q_{total}=Q_1+Q_2$. So, in comparison with just one capacitor, we need more charge for the same voltage. The total capacitance is the ratio of the total charge to the total voltage:

$$C = \frac{Q_{total}}{U_{total}} = \frac{Q_1+Q_2}{U_{total}} = \frac{Q_1}{U_{total}} + \frac{Q_2}{U_{total}} = C_1 + C_2 \quad (1)$$

For parallel connection the resulting capacitance can be found as shown below:

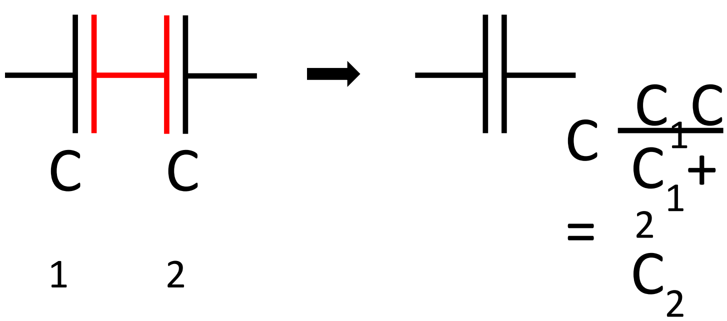


Figure 2. Series connection of the capacitors

In this case the charge induced on the corresponding plates of both capacitors is the same. We can understand that by considering the part of the circuit shown in red. This part includes one plate of the capacitor C_1 which is connected to a plate of the capacitor C_2. After we connect the capacitors to a voltage source, the total charge of this part will remain zero, since it is disconnected from the rest of the circuit and it has had no charge before we connected the battery. So the charges on "inner" (red, Figure 2) plates on the capacitors C1 and C2 are equal in magnitude but have opposite signs. The charges on the "outer" plates of the capacitors are equal to those on the corresponding "inner" plates: $Q_{total}=Q_1=Q_2$. The total voltage on two capacitors U connected in series is the sum of the voltages on each capacitor: $U_{total}=U_1+U_2$. We can express the total voltages on the capacitors as

$$U_{total} = U_1 + U_2 = \frac{Q_{total}}{C_1} + \frac{Q_{total}}{C_2} = Q_{total}\left(\frac{1}{C_1} + \frac{1}{C_2}\right) = \frac{Q_{total}}{C}\,, \qquad (2)$$

where

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}, \qquad \text{or} \qquad C = \frac{C_1 C_2}{C_1 + C_2} \qquad (3)$$

Problems:

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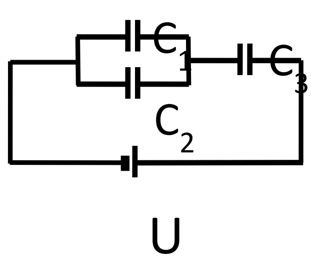
1. You connected a capacitor to a battery. After a short time the current in the circuit is zero. Then you started increasing the distance between the plates. Will the current be zero while you are increasing the distance? Explain your answer.

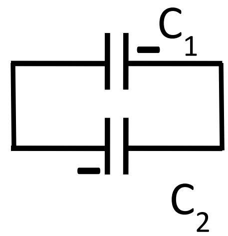
2. Calculate the capacitance of a plane capacitor if the area of the plate is 6cm^2 and the distance between the plates is 1mm. Assume that the plates are in vacuum

3. Find the charge on each capacitor at the circuit below:



   Assume that you know capacitance of each capacitor and the voltage of the battery (U). (*remember that if two circuit elements are connected in parallel then their voltages are equal; if two elements are connected in series then the total voltage is the sum of the individual voltages*)

4. Two capacitors have charges q_1 and q_2 and capacitances C_1 and C_2. We are connecting the plates of one capacitor to the plates of the other as shown in the Figure below:



Find the charges on the capacitors after the connection.