Homework 16

N1

$$\begin{cases} E_{mech,1} = E_{kin,1} + E_{pot,1} \\ E_{mech,2} = E_{kin,2} + E_{pot,2} \end{cases}$$

$$\Delta E_{mech} = E_{mech,2} - E_{mech,1} =$$

$$= \Delta E_{kin} + \Delta E_{pot} = 0$$

$$\Rightarrow \quad \boxed{\Delta E_{kin} = -\Delta E_{pot}}$$

N2.

$$\Delta E_{pot} = -2\,J$$

$$h = -\frac{\Delta E_{pot}}{mg} \simeq 4\,m$$

N3. $E_{pot} = MgH$, $H = 100\,m$

$$V = 10\,km^3 = 10\,(10^3 m)^3 = 10^{10}\,m^3$$

$\underline{N3.}$ $M = 1000 \frac{kg}{m^3} \cdot 10^{10} m^3 = 10^{13} kg$

$$E_{pot} \simeq 10^{13} kg \cdot 10 \frac{m}{s^2} \cdot 100 m$$

$$\simeq 10^{16} J.$$

Annual el. en. cons $\simeq 10^4$ kWh/year

$\simeq 3.6 \cdot 10^{10}$ J/year

$$N_y = \frac{10^{16} J}{3.6 \cdot 10^{10} J/year} \simeq 300{,}000 \text{ years}$$

$\underline{N4.}^*$

M $E_{pot} = ?$

$$E_{pot} = \underbrace{\sum_{i=1}^{N} m_i \cdot h_i}\cdot g$$

Center of mass: $R_{cm} = \frac{\sum_{i=1}^{N} m_i h_i}{\sum_{i=1} m_i}$

$M = \sum_{i=1}^{N} m_i$

$$\boxed{E_{pot} = M \cdot R_{cm} \cdot g = Mg\frac{L}{2}}$$

Classwork   Work and energy

Previously: $E_{pot} = mgh$

$$E_{mech.} = E_{kin} + E_{pot.} = \text{const.}$$

no forces other than gravity

Another conserved quantity: momentum

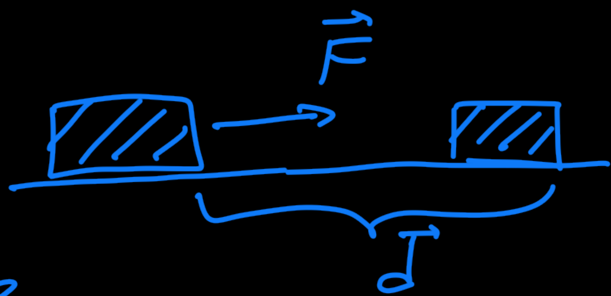
For the momentum to change: one needs an external force!

$$\boxed{\Delta \vec{p} = \vec{J} = \vec{F} \cdot t}$$

Similarly, the change in the kinetic energy is related to the force (or forces):

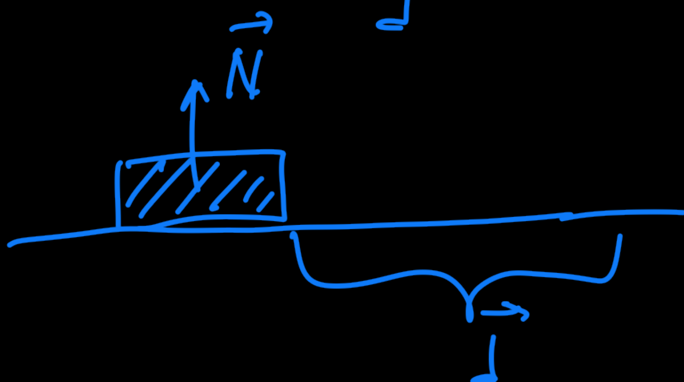
$$\boxed{\Delta E_{kin} = W = F \cdot d}$$

Work = distance × force along the direction of motion.



$\Rightarrow \boxed{W = +F \cdot d > 0}$

$\Rightarrow \boxed{W = -F \cdot d < 0}$

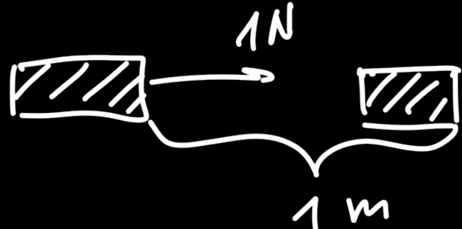


$\Rightarrow \boxed{W_N = 0}$

$$[W] = 1\,N \cdot 1\,m = 1\,kg \cdot \frac{m}{s^2} \cdot 1\,m =$$

$$= 1\,kg\,\frac{m^2}{s^2} = 1\,J$$

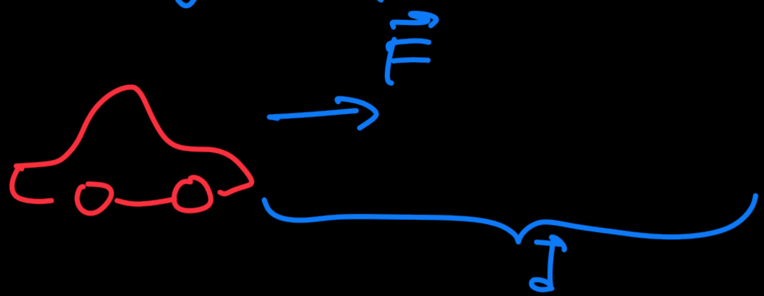
$$= [E_{kin}] \checkmark$$



Ex 1.

A car is accelerating from rest up to $30\ \text{m/s}$. $\boxed{M = 2000\ \text{kg}}$.

What work was done by the engine?



$$\Delta E_{kin} = W = F \cdot d > 0$$

$$\Delta E_{kin} = \underbrace{\frac{m v^2}{2}}_{E_{kin,after}} - \cancel{E_{kin,before}}^{\,0}$$

$$= \frac{2000\,\text{kg} \cdot 900\ \text{m}^2/\text{s}^2}{2} = 900\ \text{kJ}$$

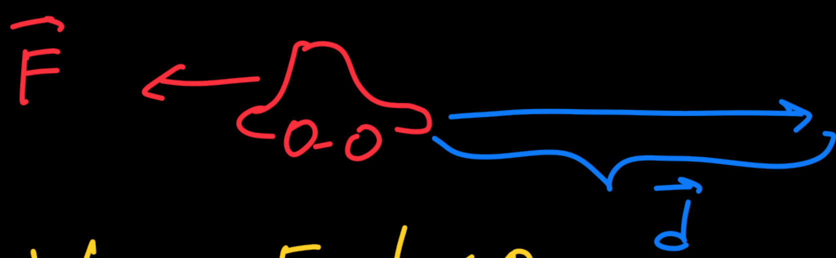
$$\boxed{W = \Delta E_{kin} = 900\ \text{kJ}}$$

Ex.2. if $F = 3\,kN$, what is d?

$$W = +F \cdot d \Rightarrow d = \frac{W}{F}$$

$$\boxed{d = \frac{900\,kJ}{3\,kN} = 300\,m}$$

Deceleration:

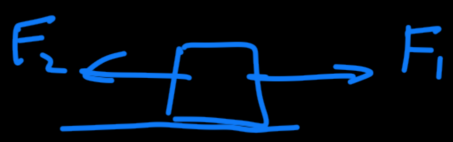


$$W = -F \cdot d < 0$$

$$\Rightarrow \Delta E_{kin} < 0.$$

What about the change in the mechanical energy?

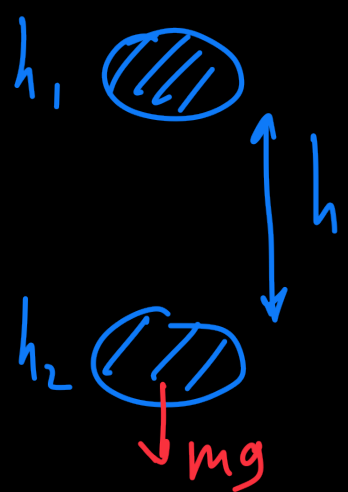
$$\boxed{\Delta E_{kin} = W_{all\ forces}} \quad (1)$$



$$\boxed{W_{all\ forces} = W_1 + W_2}$$

Work done by the gravity force

$$\boxed{h_1 - h_2 = h}$$



$$W_{grav} = mg \cdot h$$

$$\boxed{\Delta E_{kin.} = mgh}$$

$$\boxed{\Delta E_{kin} = -\Delta E_{pot}.}$$

$$\Delta E_{pot} = mgh_2 - mgh_1 = -mgh$$

$$\boxed{W_{grav} = -\Delta E_{pot}.}$$

Work-energy Theorem

$$\Delta E_{kin} = W_{all\ forces}$$

$$= W_{grav.} + W_{except\ grav.}$$

$$\boxed{W_{grav.} = -\Delta E_{pot}}$$

$$\Delta E_{kin} - W_{grav} = W_{except\ grav.}$$

$$\boxed{\Delta E_{kin} + \Delta E_{pot} = W_{except\ grav.}}$$

$$\boxed{\Delta E_{mech.} = W_{except\ grav.}} \quad ②$$

① and ② are equivalent!