

Homework 13

Mechanical work

We learned that in order to change kinetic energy of an object we have to change the absolute value of its velocity (the speed). To do that we need acceleration and, hence, **force** (we know very well that there is no acceleration without force). But it is not enough just to apply force. We also know that there is acceleration which just changes the direction of the velocity but leaves its absolute value unaltered. For example when you ride on merry-go-round your speed is constant as well as kinetic energy. But this is accelerated motion since the direction of your motion is changing permanently. To maintain this kind of motion we also need force (this force is called *centripetal force* – we will discuss it later).

To change kinetic energy of an object we should have the **displacement** of the object **along the direction of the force**. If there are both force applied to an object and the displacement of the object along the force a physicist says that **the force performs work on the object**. The “everyday” meaning of the word “work” is different (for example, a swimming pool guard may spend the whole working day watching the swimming people – there is no force, and, sometimes, no much displacement☺). In physics the work is:

Work = Force x Displacement, parallel to the force

$$A=Fs$$

To change kinetic energy of an object we have to apply force which will perform work on the object. In spite of the force and the displacement are vectors, the work is a scalar – it has no direction, just magnitude.

The work neither creates nor destroys the energy. This is impossible. But the work is the tool which converts the energy from one form to another and/or transfers the energy from one object to another.

The unit for the work measurement in the International System is Joule (J) – same as for the energy.

The work can be positive and negative. When the object moves along the applied total net force the work is positive and the kinetic energy of an object increases. Sometimes the force applied against the displacement of an object – in this case the object decelerates, the work of the force is negative in this case and the kinetic energy decreases.

To change the kinetic energy the object work has to be done!

The change of the kinetic energy of an object is equal to the total net work performed on the object. This sentence is referred to as “kinetic energy theorem”.

Example 1: We push a cart of the mass ***m*** with the force ***F***. At the first moment the cart was at rest. Find the kinetic energy of the cart after it passes distance ***d***.

A possible way of solution:

1. Let us find the acceleration of the cart: $a = \frac{F}{m}$

2. Let us find the time t which is necessary to cover the distance d :

$$d = \frac{at^2}{2}, \text{ so we can find } t: t = \sqrt{\frac{2d}{a}} = \sqrt{\frac{2dm}{F}}$$

3. Let us find the speed of the cart in the time t : $v = at = \frac{F}{m} \cdot \sqrt{\frac{2dm}{F}}$

4. Finally, let us calculate the kinetic energy:

$$E_k = \frac{mv^2}{2} = \frac{1}{2} \cdot m \cdot \left(\frac{F}{m}\right)^2 \cdot \left(\sqrt{\frac{2dm}{F}}\right)^2 = \frac{1}{2} \cdot \frac{m \cdot F \cdot F \cdot 2 \cdot d \cdot m}{m \cdot m \cdot F} = F \cdot d !!!$$

Another possible way of solution: $E_k = F \cdot d$. This is simpler. We just have to apply the kinetic energy theorem which states that the work performed on the cart is equal to change of the cart's kinetic energy.

Example 2: At the beginning we push the cart of the mass m and it started moving at a speed of v_0 . How far it will roll if the friction force is F ?

A possible way of solution:

1. Let us find the acceleration of the cart: $a = -\frac{F}{m}$ ("minus" is because the acceleration due to friction is directed against the motion)

2. Let us find in what time t the cart will stop:

$$t = \frac{v_{final} - v_0}{a} = \frac{0 - v_0}{a} = -\frac{v_0}{a} = \frac{mv_0}{F}$$

4. Finally, let us calculate the distance d :

$$d = v_0 t - \frac{at^2}{2} = v_0 \cdot \left(\frac{mv_0}{F}\right) - \frac{1}{2} \left(-\frac{F}{m}\right) \cdot \left(\frac{mv_0}{F}\right)^2 = \frac{mv_0^2}{F} - \frac{1}{2} \frac{mv_0^2}{F} = \frac{mv_0^2}{2} \div F$$

$$d = \frac{E_K}{F}$$

Another possible way of solution:

$E_K = F \cdot d$. The friction force performs negative work on the cart and “eats” kinetic energy of the cart. I said “eats” but we know that the energy can not be destroyed. Friction just transforms the kinetic energy into another energy form – heat.

$$\text{So } d = \frac{E_K}{F} = \frac{mv_0^2}{2} \div F$$

Problems:

1. Find the work of the friction force which is necessary to stop the 1000kg car moving at a speed of 72km/h. The friction coefficient is 0.1
2. A barrel is full of water. A boy emptied half of the barrel using a bucket. A girl emptied the other half. Who performed more work: the boy or the girl? Or, may be the works were equal?
3. A water pump lifts 20 liters of water per second to the water supply tank which is 10m over the ground level. What work is performed by the pump per 1 hour?
4. An ice cube is sliding down in a bowl. The inside of the bowl is shaped as a half of the sphere with diameter of 40cm. Find the velocity of the cube at the bottom of the bowl.

