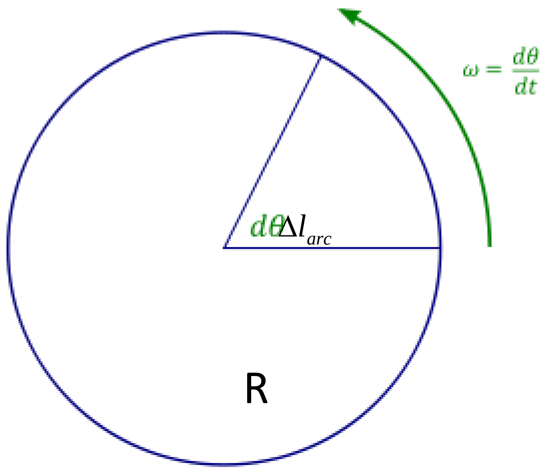
Rotation of a Solid Body

Angle (in radians): length of ark over radius

$$\Delta\theta = \frac{\Delta l}{R}$$



Angular velocity: $$\omega = \frac{\Delta\theta}{\Delta t}$$

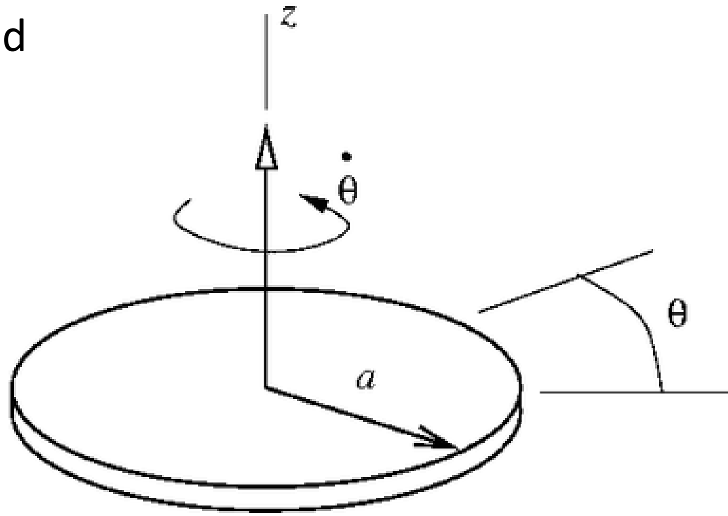
It is related to regular (linear) speed of rotational motion as:

$$v = \frac{\Delta l_{arc}}{\Delta t} = \varpi R$$

Kinetic energy of a rotating object

In a rotating rigid body, the further you are from the center, the larger is your speed!

Let's "break" a rotating object onto little pieces and add their kinetic energies together:

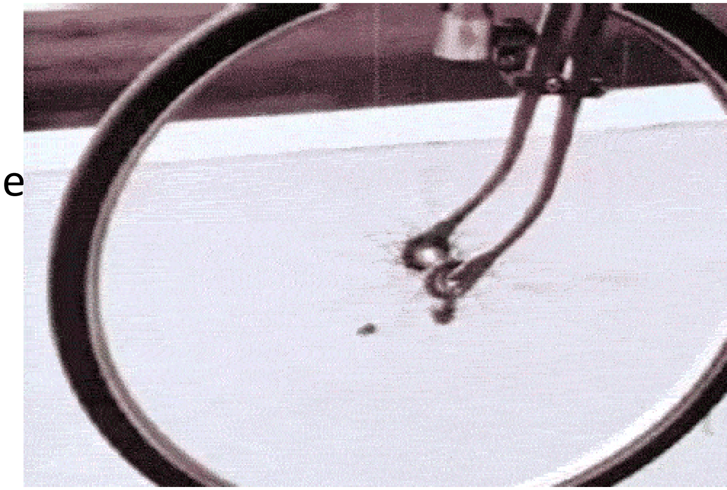


$$K = \sum_i \frac{m_i v_i^2}{2} = \sum_i \frac{m_i(\omega r_i)^2}{2} = \frac{\omega^2}{2} \sum_i m_i r_i^2$$

Here each piece has its own index ("i"), mass m_i, and speed v_i. However, they all have the same angular velocity since they are part of a rigid body.

Therefore, the formula for rotational kinetic energy is

$$K = \frac{I\omega^2}{2}$$



Here $I = \sum_i m_i r_i^2$ is called moment of inertia (r_i is the distance of piece "i" from the axis of rotation).
You can easily find moment of inertia of a thin ring (or hoop, or bicycle wheel): most of its mass M is at the same distance R from the center, so

$$I_{ring} = MR^2$$

Homework

Problem 1

a) Moment of inertia of a uniform disk of mass M and radius R is $I_{disk} = MR^2/2$. Find the total kinetic energy of a disk as it moves with linear speed V, without sliding. Note that its kinetic energy consists of regular ("translational") and rotational one.

b) The disk starts rolling down a hill without sliding, with zero initial speed. What will be its final speed on the ground level, if the hill is 10 m high , and there is no energy loss?

Problem 2

Consider a vehicle of mass M, with its wheels accounting for 50% of that mass. What well be a kinetic energy of this vehicle when it moves at peed V? Assume the wheels to be uniform disks as in problem 1.