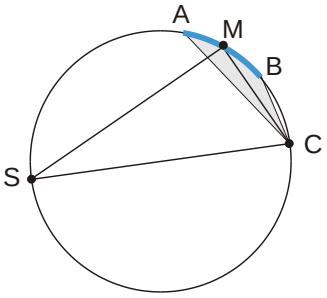
Homework for November 24, 2024 Math 9

Geometry.

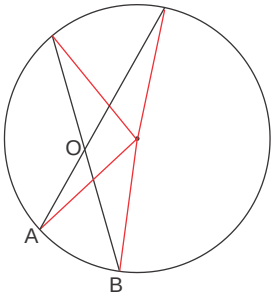
Review the previous classwork notes. Solve the following problems, including problems from the last homework (if you have not solved them yet).

Problems.

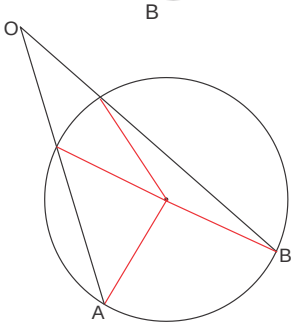
1. A **Rowland focusing** mirror is a device which focuses light of a certain color from the point source S onto a point, C, at sample. The mirror has the shape of a circular arc AB of 40 cm length. It is positioned so that its center, M, is at a distance 4 m from the point source S and at a distance 2 m from the sample C, $|SM| = 4$ m, $|MC| = 2$ m. The light ray of the color of interest is reflected so that it forms a 90˚ angle with the incident ray (e.g. angle SMC in the figure on the right is 90˚).



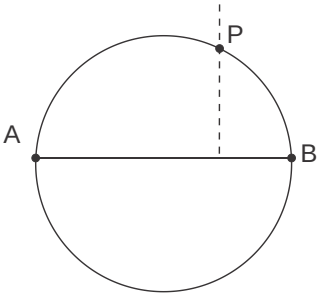
   a. What is the radius of the Rowland circle?
   b. What is the angular size of the light beam illuminating the sample (shaded angle ACB in the figure)? Does it depend on the position of sample, C?



2. Prove that an angle whose vertex lies inside a disk is measured by a semi-sum of the two arcs, one of which is intercepted by this angle, and the other by the angle vertical to it.



3. Prove that an angle whose vertex lies outside a disk and whose sides intersect the circle, is measured by a semi-difference the two intercepted arcs.
4. Given a circle and a diameter drawn of that circle, using only a straightedge, draw a perpendicular to that diameter passing through (i) point P on the circle; (ii) point P outside the circle (iii) point P inside the circle.



5. Prove that in a right triangle, each side of the right angle is the geometric mean between the hypotenuse and its projection onto the hypotenuse. That is, if BD is the altitude from the vertex of the right angle, ABC, onto the hypotenuse, AC, then $|AB|^2 = |AC||AD|$.
6. Prove that three medians in a triangle divide it into six smaller triangles of equal area.

Algebra.

Review the classwork handout and complete the exercises. Solve the remaining problems from the previous homework (you may skip the ones considered in class). Solve the following problems.

1. Using Euclid's algorithm, provide the continued fraction representation for the following numbers. Using the calculator, compare the values obtained by truncating the continued fraction at 1st, 2nd, 3rd, … level with the value of the number itself (in decimal representation).
   a. $\frac{1351}{780}$
   b. $\frac{25344}{8069}$
   c. $\frac{29376}{9347}$
   d. $\frac{6732}{1785}$
   e. $\frac{2187}{2048}$
   f. $\frac{3125}{2401}$
2. Is there a number, x, represented by the following infinite continued fraction? If so, find it.
   a. $x = 5 - \cfrac{6}{5-\cfrac{6}{5-\cfrac{6}{5-\cdots}}}$
   b. $x = 2 - \cfrac{1}{2-\cfrac{1}{2-\cfrac{1}{2-\cdots}}}$
   c. $x = 1 - \cfrac{6}{1-\cfrac{6}{1-\cfrac{6}{1-\cdots}}}$

3. Write the first few terms in the following sequence ($n \geq 1$),

$$n\ fractions \begin{cases} \cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cdots}}} \\ \qquad \ldots + \cfrac{1}{1+x} \end{cases} = ?$$

   a. Try guessing the general formula of this fraction for any n.

   b. Using mathematical induction, try proving the formula you guessed.

4. Can you prove that,

   a.

$$\frac{3+\sqrt{17}}{2} = 3 + \cfrac{2}{3+\cfrac{2}{3+\cfrac{2}{3+\cdots}}} ?$$

   b. $$1 = 3 - \cfrac{2}{3-\cfrac{2}{3-\cfrac{2}{3-\cdots}}} ?$$

   c.

$$\cfrac{4}{2+\cfrac{4}{2+\cfrac{4}{2+\cdots}}} = 1 + \cfrac{1}{4+\cfrac{1}{4+\cfrac{1}{4+\cdots}}} ?$$

   Find these numbers?