#### Baseline revue test. Algebra.

- 1. In the first quarter of the year, the retirement fund of Alex's parents lost 15% of its value, in the second quarter it lost another 5%. Then, in the third and fourth quarters it gained 10% and 20%, respectively.
  - a. What is the total net gain/loss of Alex's parents' retirement fund over the year (in percent)?
  - b. What is the mean quarterly gain/loss over the year?
- 2. Open parentheses and expand the following expressions.

a. 
$$(a+b)^2 =$$

b. 
$$(a - b)^2 =$$

c. 
$$(a+b)^3 =$$

d. 
$$(a - b)^3 =$$

3. Factor the following expressions

a. 
$$a^2 - b^2 =$$

b. 
$$a^2 + b^2 =$$

c. 
$$a^3 - b^3 =$$

d. 
$$a^3 + b^3 =$$

e. 
$$1 + a + a^2 + a^3 =$$

4. For a quadratic equation  $ax^2 + bx + c = 0$  the roots are,

$$x_{1,2} =$$

and they have the following properties,

$$x_1 + x_2 =$$

$$x_1 \cdot x_2 =$$

5. Open parentheses and expand the following expression,

$$(a+b)^{10} =$$

- 6. What is the number of permutations of n objects?
- 7. How many ways are there to select k objects out of n if,
  - a. order does matter?
  - b. order does not matter?
- 8. Write the formula for a binomial coefficient

$$C_n^k \equiv {}_n C_k \equiv {n \choose k} =$$

and explain its relation to combinatorics and certain counting problems.

9. What is the remainder of division of  $3^{2023}$  by 7?

1. Open the parentheses and expand the following expressions

a. 
$$(a+b)^2 =$$

b. 
$$(a - b)^3 =$$

2. Factor the following expressions

a. 
$$a^2 - b^2 =$$

b. 
$$a^3 - b^3 =$$

c. 
$$a^3 + b^3 =$$

d. 
$$1 + a + a^2 + a^3 =$$

3. Solve the following equation:

$$\frac{1}{1-x} + \frac{1}{1-\frac{1}{x}} = 1$$

4. Solve the inequality:

$$x + \frac{1}{x} > 3\frac{1}{3}$$

- 5. Find the remainder of  $2024^{2024}$  upon division by 3.
- 6. Eight teams have reached the quarter-finals of a soccer tournament.
  - a. How many ways are there for these teams to be paired to play the quarter-final games against each other?
  - b. How many different tournament outcomes in terms of which medal (gold, silver, bronze) goes to which team are possible?
- 7. Three equal segments are cut off a circle so that an equilateral triangle is obtained. Find the area of each of these segments if the radius of the circle is r.

8. Open the parentheses and expand the following expressions

a. 
$$(a+3b)^2 =$$

b. 
$$(a - b)^3 =$$

9. Solve the inequality

$$x + \frac{1}{x} > 4.25$$

a. 
$$a^2 - b^2 =$$

b. 
$$a^3 - b^3 =$$

c. 
$$a^3 + b^3 =$$

d. 
$$1 + a + a^2 + a^3 =$$

- 11. Alice, Bob, Charlie, and Diana are buying ice cream. The store has 15 different flavors of ice cream. How many ways there are for them to choose if each kid chooses one ice cream flavor? How will the answer change if in addition, we require that no two of them choose the same flavor?
- 12. Find the coefficient of  $x^2$  in the polynomial  $(2x 1)^8$
- 13. How many divisors does the number 360 have?
- 14. What is the last digit of of  $7^{2021}$ .
- 15. Given angle  $\angle AOB$ , construct the angle bisector using straightedge and compass. (You can do your construction online here: https://www.geogebra.org/geometry/nrn7t5nr)

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b. 
$$(a - b)^3 =$$

a. 
$$a^2 - b^2 =$$

b. 
$$a^3 - b^3 =$$

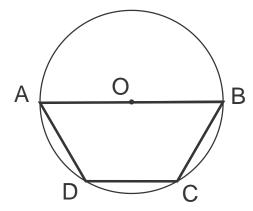
c. 
$$a^3 + b^3 =$$

d. 
$$1 + a + a^2 + a^3 =$$

- 3. Find the remainder of  $2020^{2020}$  upon division by 3.
- 4. Solve the equation

$$x + \frac{1}{x} = 4\frac{1}{4}$$

- 5. Eight teams have reached the quarter-finals of the soccer World Cup.
  - a. How many ways are there for these teams to be paired to play the quarter-final games?
  - b. How many different outcomes of which team wins which medal (gold, silver, bronze) are possible?
- 6. Trapezoid ABCD is inscribed in a circle of radius r as shown in the Figure. The longer base of the trapezoid is equal to diameter, |AB| = 2r, while the shorter bas equals r. Find the area of the trapezoid ABCD.



1. Factor the following expressions

a. 
$$a^2 - b^2 =$$

b. 
$$a^3 - b^3 =$$

c. 
$$a^3 + b^3 =$$

d. 
$$1 + a + a^2 + a^3 =$$

- 2. Find the coefficient of  $x^7$  in the polynomial  $(1 + 2x)^9$ .
- 3. Let  $x_1$ ,  $x_2$  be roots of the equation  $x^2 = x + 1$ . Find  $\frac{1}{x_1} + \frac{1}{x_2}$ .
- 4. Find the remainder of  $2^{2019}$  upon division by 7.
- 5. Let ABCD be a trapezoid, with bases AD and BC, and let E, F be midpoints of sides AB, CD respectively. If BC = 2 cm, AD = 6 cm, then what is EF? Can you prove your answer?

# Math 9 placement test 2018

6. Open brackets and expand the following expressions

a. 
$$(a+b)^2 =$$

b. 
$$(a - b)^3 =$$

7. Factor the following expressions

a. 
$$a^2 - b^2 =$$

b. 
$$a^3 - b^3 =$$

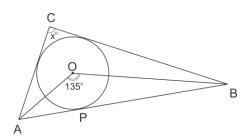
c. 
$$a^3 + b^3 =$$

d. 
$$1 + a + a^2 + a^3 =$$

8. Solve the following inequality. Write your answer as a set of possible values for x.

$$\frac{(x+2)^2(x-7)}{x+3} \le 0$$

- 9. Find the remainder of  $2^{2019}$  upon division by 7.
- 10. Let  $x_1$ ,  $x_2$  be roots of the equation  $x^2 = x + 1$ . Find  $\frac{1}{x_1} + \frac{1}{x_2}$ .
- 11. Find the remainder upon division of  $2^{2019}$  by 7.
- 12. *O* is the center of the inscribed circle in triangle *ABC*. The angle *AOB* is 135 degrees. Find the angle *ACB*.



1. Open brackets and expand the following expressions

a. 
$$(a+b)^2 =$$

b. 
$$(a-b)^3 =$$

2. Factor the following expressions

a. 
$$a^2 - b^2 =$$

b. 
$$a^3 - b^3 =$$

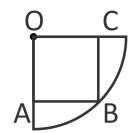
c. 
$$a^3 + b^3 =$$

d. 
$$1 + a + a^2 + a^3 =$$

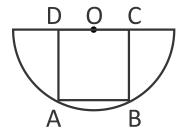
- 3. Find the remainder of  $3^{2017}$  upon division by 4.
- 4. Solve the equation

$$x + \frac{1}{x} = 7\frac{1}{7}$$

- 5. Eight teams have reached the quarter-finals of the soccer World Cup.
  - a. How many ways are there for these teams to be paired to play the quarter-final games?
  - b. How many different outcomes of which team wins which medal (gold, silver, bronze) are possible?
- 6. Find the area of a square inscribed in
  - a. a quarter circle of radius *r*, as shown in the Figure below,



b. a semicircle circle of radius r as shown in the Figure below.



1. Open brackets and expand the following expressions

a. 
$$(a+b)^2 =$$

b. 
$$(a - b)^3 =$$

a. 
$$a^2 - b^2 =$$

b. 
$$a^3 - b^3 =$$

c. 
$$a^3 + b^3 =$$

d. 
$$1 + a + a^2 + a^3 =$$

- 3. Find the remainder of  $3^{2016}$  upon division by 5.
- 4. Solve the equation

$$\frac{x^2+1}{x} - \frac{2x}{x^2+1} = 1$$

- 5. Eight teams have reached the quarter-finals of the soccer World Cup.
  - a. How many ways are there for these teams to be paired to play the quarter-final games?
  - b. How many different outcomes of which team wins which medal (gold, silver, bronze) are possible?
- 6. Four equal segments are cut off a circle of radius r so that a square is obtained. Find the area of each of these segments.

- 1. How many ways are there to choose a team captain and 6 team members out of 15 candidates?
- 2. If  $x_1, x_2$  are roots of the square equation  $x^2 + 2x 7$ , what is  $x_1x_2 ? \frac{1}{x_1} + \frac{1}{x_2} ?$
- 3. Simplify the following expression

$$\frac{(a^2 - b^2)^3}{(a - b)(a + b)^2}$$

- 4. How many "words" can you form by permuting the letters of the word "letter"? (A "word" is any combination of letters, not necessarily meaningful)
- 5. Points A = (0,0), B = (2,0), and C on the coordinate plane form an equilateral triangle. What are the coordinates of point C?
- 6. Factor  $a^4 b^4$ .
- 7. Corners of a square with the side *a* are cut off so that a regular octagon is obtained. Find the area of this octagon.
- 8. Solve the inequality

$$\frac{x+5}{x^2 - 2x - 3} > 0$$

- 9. If we write the polynomial  $(x + 2)^{10}$  in the usual form  $x^{10} + a_1x^9 + a_2x^8 + ...$ , what would be the coefficient of  $x^6$ ?
- 10. Find all integer numbers which give remainder 2 upon division by 7 and remainder 5 upon division by 13.
- 11. Given triangle *ABC*, explain how to construct (using ruler and compass) a point which is at equal distance from points *A*, *B*, and *C*.

1. Open brackets and expand the following expressions

a. 
$$(a+b)^2 =$$

b. 
$$(a - b)^3 =$$

a. 
$$a^2 - b^2 =$$

b. 
$$a^3 - b^3 =$$

c. 
$$a^3 + b^3 =$$

d. 
$$1 + a + a^2 + a^3 =$$

- 3. Find the coefficient of  $x^5$  in the expression  $(1 + 2x)^8$
- 4. Find the remainder of  $3^{2014}$  upon division by 7.
- 5. Solve the equation

$$\frac{x^2+1}{2x} + \frac{2x}{x^2+1} = 2$$

- 6. Eight teams have reached the quarter-finals of the soccer World Cup.
  - a. How many ways are there for these teams to be paired to play the quarter-final games?
  - b. How many different outcomes of which team wins which medal (gold, silver, bronze) are possible?
- 7. Corners of a square with the side *a* are cut off so that a regular octagon is obtained. Find the area of this octagon.