MATH 8B [2024 NOV 10]
HANDOUT 8 : LOGIC 3 : SR FLIP-FLOP. CONDITIONALS.

SR Flip-Flop

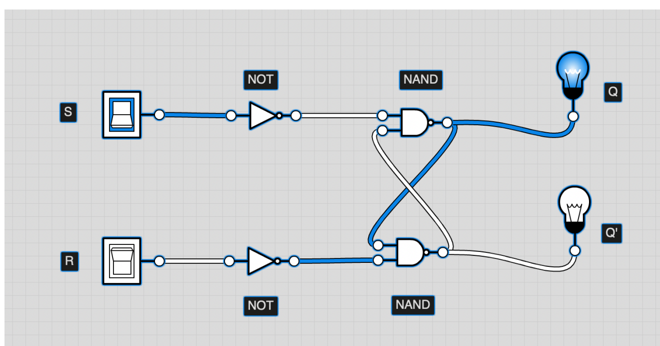
Today we will play a little more with circuits. So far we have dealt with *combinational circuits*. Such a circuit C takes in some inputs, uses some combination of gate, and implements a logic function $f_C(x_1, \ldots, x_n)$ purely of the inputs $x_1, \ldots, x_n$. (More generally, it may have multiple outputs corresponding to m Boolean functions $f_{C,1}, \ldots, f_{C,m}$.)

Now, we're going to look at a slighly more complicated type of circuit - a *sequential circuit*, for which the output does not only depend on the inputs, but also on the previous inputs (or state of the system). These are said to have memory.

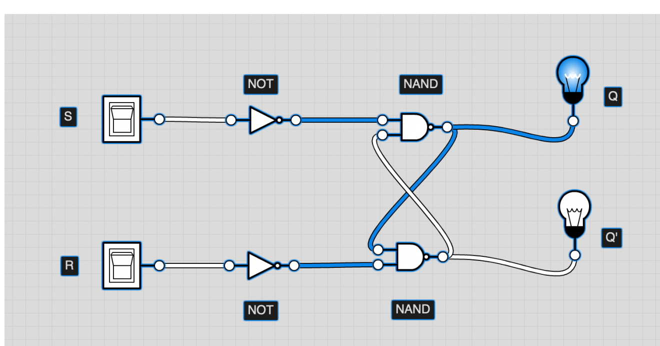
A simple example of such a circuit is the following, which is called SR Flip-Flop. Interestingly, the output of NAND-gates is also an input to other NAND-gates. Let us look at how it works.

Let's imagine that we turn S on to 1. The NOT-gate changes it to 0, and when it is fed to the NAND gate, the output of it would be 1, since $0 \text{ NAND } X = 1$ for any X (make sure you understand why it is so!). As a result, the Q bulb will turn on.

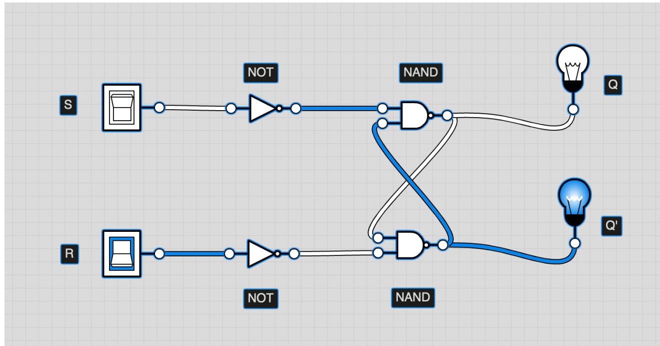
At the same time, R is off (is 0), and it is changed to 1 by the NOT-gate, and fed to NAND along with the output of the top NAND-gate, so the output of the bottom NAND-gate is 0 (since $1 \text{ NAND } 1 = 0$).



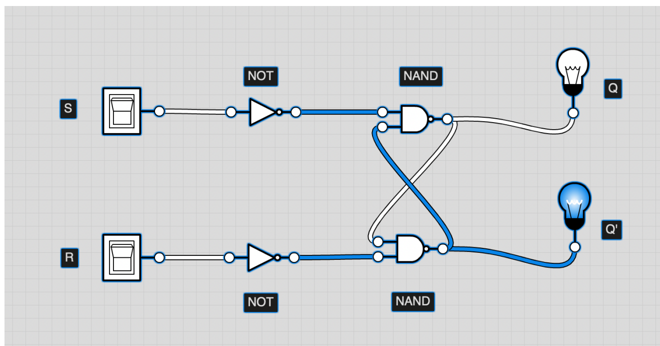
Interestingly, if we now flip S back off, the lightbulbs will not change their state: lightbulb Q will stay on, and Q' will stay off: one of the inputs to the top NAND will always stay off, regardless of what S is.



Now if we switch S to off, and turn R on, the lightbulbs will flip: Q' will be lit up, and Q will be off.



We will also observe a similar situation: now switching R to off will not change the state of lightbulbs:



The state when the top lightbulb is lit up is called S-state, that is the flip-flop is in SET position. When the lower lightbulb is on, the flip-flop is in RESET (R) position.

A simple way to summarize the working of this circuit is: when the reset R is on, then toggling S will flip the state of Q (to be the same as S). So it allows us to reset the main output (the set state Q). When the reset R is off, then Q turns on (if it not already on) as soon as the set input S comes on, and stays on after that, until we reset again.

The interesting thing about this circuit is that it has **memory**: once it's in the SET-state, the top lightbulb indicates it, and switching S-switch off won't change anything – the top lightbulb will still be on. Similarly, once we're in R-state, we can switch the R-switch off, but the lightbulbs will still indicate that we are in the R-state (the 2nd light bulb is on)

Review: Implication and Equivalence

Recall the implication or conditional operator $\Rightarrow$, which has the truth table

A	B	$A \Rightarrow B$
T	T	T
T	F	F
F	T	T
F	F	T

Note that in particular, in all situations where A is false, $A \Rightarrow B$ is automatically true. E.g., a statement "if $2 \times 2 = 5$, then..." is automatically true, no matter what conclusion one puts in place of dots.

We showed that it is equivalent to $\neg A \vee B$.

We also looked that the equivalence operator $\Leftrightarrow$, defined as: $(A \Leftrightarrow B)$ is true if A, B always have the same value (both true or both false). We showed last homework that it is the same as $(A \Rightarrow B)\neg(B \Rightarrow A)$. We also saw that it is the same as $\neg(A \text{ XOR } B)$.

PROBLEMS

1. Show that $A \Rightarrow B$ is not equivalent to $B \Rightarrow A$, i.e. they have different truth tables and one of them can be true while the other is false.

2. Prove the contrapositive law: $A \Rightarrow B$ is equivalent to $(\neg B) \Rightarrow (\neg A)$

3. Can you rewrite $\neg(A \Rightarrow B)$ without using the implication operation?

4. Consider the following statement (from a parent to his son):

"If you do not clean your room, you can't go to the movies"

Is it the same as:

(a) Clean your room, or you can't go to the movies
(b) You must clean your room to go to the movies
(c) If you clean your room, you can go to the movies

5. English language (and in particular, mathematical English) has a number of ways to say the same thing. Can you rewrite each of the verbal statements below using basic logic operations (including implications), and variables

- A: you get score of 90 or above on the final exam
- B: you get an A grade for the class

(As you will realize, many of these statements are in fact equivalent)

(a) To get A for the class, it is required that you get 90 or higher on the midterm
(b) To get A for the class, it is sufficient that you get 90 or higher on the midterm
(c) You can't get A for the class unless you got 90 or above on the final exam
(d) To get A for the class, it is necessary and sufficient that you get 90 or higher on the midterm

6. Show that in all situations where A is true and $A \Rightarrow B$ is true, B must also be true. [This simple rule has a name: it is called *Modus Ponens*.]

7. Show that if $A \Rightarrow B$ is true, and B is false, then A must be false. [This is called *Modus Tollens*.]

8. Is $A \Rightarrow (B \Rightarrow C)$ the same as $(A \Rightarrow B) \Rightarrow C$?

9. Show that $A \Rightarrow (B \Rightarrow C)$ the same as $(A \wedge B) \Rightarrow C$.

10. Fill in the following table with True/False values:

	Commutative	Associative
AND		
OR		
XOR		
NAND		
$\Rightarrow$		
$\Leftrightarrow$		